SUPERVISOR TO ATTACH PROCESSING LABEL HERE

VCE Mathematical Methods
Units 3 & 4
Practice examination

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- In questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(24 marks)

1 Let $f(x)=3x+2$ and $g(x)=-3x+5$.
The expression $f(g(x))-g(f(x))$ is equal to

A $16-18x$
B 0
C 18
D -18
E $18-9x$

(1 mark)

2 A differentiable function f satisfies $f'(x)=(x-1)(x+1)$. The stationary point of f at $x=1$ is

A unable to be classified from f'
B a local maximum
C a local minimum
D a stationary point of inflection
E not a stationary point

(1 mark)

3 Given that $\frac{d}{dx}(3x\cos(x))=-3x\sin(x)+3\cos(x)$, the indefinite integral $\int -3x\sin(x)\,dx$ is equal to

A $-3x\sin(x)+3\cos(x)+c$
B $3x\cos(x)+3\sin(x)+c$
C $-3x\cos(x)+3\sin(x)+c$
D $3x\cos(x)+c$
E $3x\cos(x)-3\sin(x)+c$

(1 mark)

4 A bag has 9 red tokens and 3 blue tokens. Three are drawn without replacement. The probability of red, blue, red, in that order, is

A $\frac{27}{55}$
B $\frac{9}{64}$
C $\frac{81}{440}$
D $\frac{1}{8}$
E $\frac{9}{55}$

(1 mark)

5 The function $f(x)=-5+\frac{4}{(x-2)^2}$ is not continuous at $x=$

A -5
B 4
C -2
D 2

E 0

(1 mark)

6 A graph is transformed first by T_1 and then by T_2, where

$T_1 : \ (x,y) \mapsto (2x-1, 5y+2)$ and $T_2 : \ (x,y) \mapsto (4x+4, 3y-3)$.

The single point mapping equivalent to this composition is

A $(x,y) \mapsto (8x+3, 15y-1)$

B $(x,y) \mapsto (8x+8, 15y-9)$

C $(x,y) \mapsto (8x+7, 15y-13)$

D $(x,y) \mapsto (8x, 15y+3)$

E $(x,y) \mapsto (20x, 6y+3)$

(1 mark)

7 Let $g(x) = af(2(x-h)) + k$. Under this transformation, the points $(8,2)$ and $(-8,-3)$ on $y = f(x)$ are mapped to $(1,8)$ and $(-7,-2)$, respectively.

The ordered triple (a,h,k) is

A $\left(\frac{1}{2}, -3, 4\right)$

B $(2,-3,-4)$

C $(2,-3,4)$

D $(-2,-3,4)$

E $(2,3,4)$

(1 mark)

8 Let $g(x) = 4f(5(x+3)) + 3$ and $q(x) = 3f(2(x-1)) + 4$.

A point on the graph of $y = g(x)$ is mapped to the corresponding point on the graph of $y = q(x)$ by

A $(x,y) \mapsto \left(\frac{5}{2}(x+3)+1, \frac{3}{4}(y-3)+4\right)$

B $(x,y) \mapsto \left(\frac{2}{5}(x+3)+1, \frac{4}{3}(y-3)+4\right)$

C $(x,y) \mapsto \left(\frac{5}{2}(x-3)+1, \frac{3}{4}(y+3)+4\right)$

D $(x,y) \mapsto \left(\frac{2}{5}(x-1)-3, \frac{4}{3}(y-4)+3\right)$

E $(x,y) \mapsto \left(\frac{5}{2}x+4, \frac{3}{4}y+1\right)$

(1 mark)

9 The point mapping T is defined by

$T : \ (x,y) \mapsto (2x-4, 3y+4)$.

The unique fixed point of T is

A $(4,-2)$

B $(-4,4)$

C $\left(-\frac{4}{3}, 1\right)$

D $(-2,4)$

E $(-4,2)$

(1 mark)

10 The x-intercepts of the graph of $g(x) = 4[5(x-4)]^2 - 16$ are

A $\left(\frac{18}{5},0\right), \quad \left(\frac{22}{5},0\right)$
B $(-6,0), \quad (14,0)$
C $\left(-\frac{22}{5},0\right), \quad \left(-\frac{18}{5},0\right)$
D $\left(\frac{39}{10},0\right), \quad \left(\frac{41}{10},0\right)$
E $(2,0), \quad (6,0)$

(1 mark)

11 The indefinite integral $\int \frac{9x}{x^2+8}\,dx$ is equal to

A $\frac{9}{2}\log_e\left(x^2+8\right)+c$
B $9\log_e\left(x^2+8\right)+c$
C $\frac{9}{2}\log_e(x)+c$
D $18\log_e\left(x^2+8\right)+c$
E $\log_e\left(x^2+8\right)+c$

(1 mark)

12 Let X be the number of successes in n independent trials, where the probability of success on each trial is $p=\frac{1}{5}$. If $\Pr(X \geq 3) \geq \frac{2}{3}$, then the smallest possible value of n is

A 5
B 16
C 17
D 18
E 15

(1 mark)

13 Let $f(x)=x^3-x$. The tangent to the graph of f at $x=4$ has horizontal-axis intercept $(b,0)$. The value of b is

A $-\frac{60}{47}$
B 4
C $\frac{60}{47}$
D $\frac{128}{47}$
E $\frac{128}{49}$

(1 mark)

14 Let $f(x)=3-\frac{1}{x-1}$ and let $g=f^{-1}$.
The asymptotes of the graph of $y=g(x)$ are

A $x=-3, \quad y=1$
B $x=3, \quad y=-1$
C $x=1, \quad y=3$
D $x=3, \quad y=1$
E $x=-3, \quad y=-1$

(1 mark)

15 Let $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 2x + \sin(3x)$. The minimum possible value of the gradient of the graph of f is

A -3

B 1

C 5

D 2

E -1

(1 mark)

16 The average value of the function with rule $f(x) = x^2 + 1$ over the interval $[-2, 1]$ is

A $\frac{5}{4}$

B $\frac{2}{3}$

C 6

D 2

E $\frac{7}{2}$

(1 mark)

17 A fair coin is tossed five times. The probability of obtaining exactly one head, given that at least one head is obtained, is

A $\frac{1}{31}$

B $\frac{5}{27}$

C $\frac{5}{31}$

D $\frac{5}{32}$

E $\frac{4}{31}$

(1 mark)

18 Let $f(x) = \frac{(x-a)(x-4)}{(x-3)(x-1)}$, where $a \in \mathbb{R}$. The discontinuity at $x = 3$ is removable, rather than a vertical asymptote, when

A $a = -3$

B $a = 0$

C $a = 4$

D $a = 3$

E $a = 1$

(1 mark)

19 A signal is modelled by $s(t) = \sin\left(\frac{\pi t}{2}\right) + \sin\left(\frac{\pi t}{8}\right)$. Its instantaneous rate of change at $t = 0$ is

A $\frac{5}{8}$

B $\frac{\pi}{2}$

C $-\frac{5\pi}{8}$

D $\frac{\pi}{8}$

E $\frac{5\pi}{8}$

(1 mark)

20 The indefinite integral $\int \frac{4x-5}{x}\,dx$, for $x>0$, is equal to

A $4x-5\log_e(x)+c$

B $4-5\log_e(x)+c$

C $(4x-5)\log_e(x)+c$

D $4x-\frac{5}{x}+c$

E $2x^2-5\log_e(x)+c$

(1 mark)

21 Let $f(x)=\frac{5-5x}{a+3x}$, where $a\in\mathbb{R}$. The graph of f has a vertical asymptote at $x=3$ when

A $a=-9$

B $a=1$

C $a=9$

D $a=-1$

E $a=-3$

(1 mark)

22 A fair six-sided die is rolled 20 times. Let p be the probability function for the number of times the die shows 1, and let q be the probability function for the number of times the die does not show 1. Then $q(w)$ is given by

A $p(w-20)$

B $p(20-w)$

C $p\left(\frac{w}{20}\right)$

D $1-p(w)$

E $p\left(1-\frac{w}{20}\right)$

(1 mark)

23 The function f is differentiable and $f'\left(\frac{\sqrt{2}}{2}\right)=4$. Let $h(x)=f(\sin(2x))$. The value of $h'\left(\frac{\pi}{8}\right)$ is

A $4\sqrt{2}$

B $-4\sqrt{2}$

C 4

D 8

E $2\sqrt{2}$

(1 mark)

24 The graph of $f(x)=\frac{a}{x+4}+4$ passes through $(1,1)$. The value of a is

A $a=2$

B $a=-\frac{5}{3}$

C $a=15$

D $a=-15$

E $a=-\frac{3}{5}$

(1 mark)

Question 2

A workshop records how long each repair takes. Let X be the repair time, in hours, with probability density function

$$f(x) = \begin{cases} kx^2(5-x) & 0 \le x \le 5, \\ 0 & \text{elsewhere,} \end{cases}$$

where k is a real constant.

a Show that $k = \frac{12}{625}$. (2 marks)

b Find the probability that the repair time is less than 3 hours. Give your answer correct to four decimal places. (2 marks)

c Find $E(X)$, the mean repair time. (2 marks)

d Find the standard deviation of X. Give your answer correct to two decimal places. (2 marks)

e Find the median repair time, m, where $\Pr(X < m) = 0.5$. Give your answer correct to two decimal places. (2 marks)

Question 3

A patient is given a single dose of an antibiotic. The concentration of the drug in the bloodstream of the patient, in milligrams per litre, is modelled by

$$c(t) = 5te^{-0.4t}, \quad t \geq 0,$$

where t is the time in hours after the dose is given.

a State the concentration of the drug in the bloodstream of the patient when $t = 0$.(1 mark)

b Find the time at which the concentration is greatest, and find that greatest concentration. Give both answers correct to two decimal places. (3 marks)

c Find the average concentration over the first 8 hours. Give your answer correct to two decimal places. (3 marks)

d Find the rate of change of the concentration when $t = 3$. Give your answer correct to two decimal places. (2 marks)

e After the concentration has peaked it falls. Find the time at which the concentration next falls to 3 milligrams per litre. Give your answer correct to two decimal places. (2 marks) f State the concentration that the model approaches over time. (1 mark)

Question 4

A grower is digging a drainage trench along a row of vines. The cross-section of the trench is modelled by

$$d(x) = 1 - x^2,$$

where d is the depth below ground level in metres and x is the horizontal distance in metres from the centre of the trench, for $d(x) \geq 0$.

a Find the width of the trench at ground level. Give your answer in metres correct to two decimal places. (2 marks)

b State the greatest depth of the trench. (1 mark)

c Find the area of the cross-section of the trench. Give your answer in square metres correct to two decimal places. (3 marks)

d Find the average depth of the trench across its width. Give your answer in metres correct to two decimal places. (2 marks) e The trench is 40 metres long and has the same cross-section along its whole length. Find the volume of soil removed to dig it. Give your answer in cubic metres correct to two decimal places. (2 marks)

Question 5

a The population of rabbits can be modelled by the rule $r(t) = 1900\sin\left(\frac{\pi t}{70}\right) + 2800$.

i State the initial population of rabbits. (1 mark)

ii State the minimum and maximum population of rabbits. (1 mark)

iii State the number of weeks between maximum populations of rabbits. (1 mark)

b One point of minimum fox population, $(30, 700)$, and one point of maximum fox population, $(100, 2300)$, are recorded. The population of foxes can be modelled by the rule $f(t) = a\sin(b(t-65)) + 1500$.

i Show that $a = 800$ and $b = \frac{\pi}{70}$. (2 marks)

ii Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number. (1 mark) iii What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum? (1 mark)

c The population of foxes is better modelled by the transformation of $y = \sin(t)$ under Q given by

$$Q : R^2 \to R^2,\ Q\left(\begin{bmatrix} t \\ y \end{bmatrix}\right) = \begin{bmatrix} \frac{90}{\pi} & 0 \\ 0 & 800 \end{bmatrix}\begin{bmatrix} t \\ y \end{bmatrix} + \begin{bmatrix} 65 \\ 1500 \end{bmatrix}.$$

Find the average population during the first 300 weeks for the combined population of foxes and rabbits, where the population of foxes is modelled by the transformation of $y = \sin(t)$ under Q. Give your answer correct to the nearest whole number. (4 marks)

d Over a longer period of time, it is found that the increase and decrease in the population of rabbits gets smaller and smaller. The population of rabbits over a longer period of time can be modelled by the rule

$$s(t) = 1900e^{-0.004t}\sin\left(\frac{\pi t}{70}\right) + 2800, \quad \text{for all } t \geq 0.$$

i Find the average rate of change between the first two times when the population of rabbits is at a maximum. Give your answer correct to one decimal place. (2 marks)

ii Find the time, where $t > 50$, in weeks, when the rate of change of the rabbit population is at its greatest positive value. Give your answer correct to the nearest whole number. (2 marks)

iii Over time, the rabbit population approaches a particular value. State this value.(1 mark)

Question 6

An orchard packs apples into trays as they are picked. The mass of a randomly chosen apple from the packing line is a normally distributed random variable X, with a mean of 500 grams and a standard deviation of 10 grams.

a Find the probability that a randomly chosen apple has a mass greater than 515 grams. Give your answer correct to four decimal places. (1 mark)

b Find the probability that a randomly chosen apple has a mass greater than 515 grams, given that its mass is greater than 495 grams. Give your answer correct to four decimal places. (2 marks)

c Find the mass m for which $\Pr(X < m) = 0.90$. Give your answer in grams correct to one decimal place. (2 marks)

d A random sample of 200 apples is taken from the packing line. Let $\hat{P}$ be the random variable representing the proportion of apples in the sample that are underweight. Is $\hat{P}$ discrete or continuous? Justify your answer. (1 mark)

e In that sample, 16 of the apples are underweight. Find an approximate 95% confidence interval for the proportion of underweight apples produced by the packing line. Give your answer correct to three decimal places. (2 marks) f How many apples would need to be sampled to halve the width of the confidence interval found above? (1 mark)

g A crate holds 8 apples chosen at random from the packing line. Using the sample proportion above as the probability that a apple is underweight, find the probability that at least 2 of them are underweight. Give your answer correct to four decimal places. (3 marks)

Solutions

1. (1) **C.** $f(g(x)) = 17 - 9x$ and $g(f(x)) = -9x - 1$. Both have gradient -9, so the x terms cancel and the difference is the constant 18.

   (2) **C.** At $x = 1$ the derivative changes from negative to positive, so the stationary point is a local minimum.

   (3) **E.** Integrating both sides of the given derivative gives

   $$3x\cos(x) = \int 3\cos(x)\,dx + \int -3x\sin(x)\,dx$$

   , so the integral wanted is $3x\cos(x)$ minus $\int 3\cos(x)\,dx$.

   (4) **E.** Along the red-blue-red path, both the total and the available red count decrease: $\frac{9}{12}\frac{3}{11}\frac{8}{10} = \frac{9}{55}$.

   (5) **D.** The denominator is zero when $x = 2$. This value is excluded from the domain, so f is not continuous there.

   (6) **D.** Substitute the coordinates produced by T_1 into T_2. The second dilation therefore multiplies both the first coordinate and its first translation, giving $T_2 \circ T_1$: $(x, y) \mapsto (8x, 15y + 3)$.

   (7) **C.** The point mapping is $(x, y) \mapsto \left(\frac{x}{2} + h, ay + k\right)$. The horizontal coordinates determine $h = -3$. Subtracting the two vertical-coordinate equations gives $a = 2$; substitution then gives $k = 4$. Hence the ordered triple is $(2, -3, 4)$.

   (8) **A.** Recover the coordinates on $y = f(x)$ from a point on $y = g(x)$, then apply the transformation defining q. This gives $X = \frac{5}{2}(x+3) + 1$ and $Y = \frac{3}{4}(y-3) + 4$, so the mapping is

   $$(x, y) \mapsto \left(\frac{5}{2}(x+3) + 1, \frac{3}{4}(y-3) + 4\right)$$

   .

   (9) **A.** At a fixed point, $x = 2x - 4$ and $y = 3y + 4$. Solving these two independent linear equations gives $(4, -2)$.

(10) **A.** Setting $g(x)=0$ and dividing by 4 gives $[5(x-4)]^2=4$. Hence $x=4\pm\frac{2}{5}$, giving $\left(\frac{18}{5},0\right)$, $\left(\frac{22}{5},0\right)$.

(11) **A.** The numerator is $\dfrac{9}{2}$ times the derivative of the denominator, so the integral is that multiple of $\log_e$ of the denominator.

(12) **C.** Evaluating the binomial upper tail gives $\Pr(X\geq 3)<\frac{2}{3}$ when $n=16$, but $\Pr(X\geq 3)\geq\frac{2}{3}$ when $n=17$.

(13) **D.** Here $f(4)=60$ and $f'(4)=47$. Setting $y=0$ in $y-f(4)=f'(4)(x-4)$ gives $b=\frac{128}{47}$.

(14) **D.** The graph of f has asymptotes $x=1$ and $y=3$. The graph of its inverse is the reflection of this graph in $y=x$, which interchanges vertical and horizontal lines. Thus the inverse has asymptotes $x=3$ and $y=1$.

(15) **E.** $f'(x)=3\cos(3x)+2$. Since $-1\leq\cos(3x)\leq 1$, the minimum gradient is -1.

(16) **D.** The average value is $\dfrac{1}{3}\displaystyle\int_{-2}^{1}\left(x^2+1\right)dx=\dfrac{6}{3}=2$.

(17) **C.** There are $2^5-1=31$ outcomes with at least one head and $\binom{5}{1}=5$ of these have exactly one head.

(18) **D.** The denominator contains the factor $x-3$. For the discontinuity to be removable, the numerator must contain the same factor. Since its other zero is not 3, this requires $a=3$.

(19) **E.**

$$s'(t)=\frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)+\frac{\pi}{8}\cos\left(\frac{\pi t}{8}\right)$$

. At $t=0$ both cosine factors are 1, so the rate is $\frac{5\pi}{8}$.

(20) **A.** Divide first: $\dfrac{4x-5}{x}=4-5\times\dfrac{1}{x}$, and only the second piece integrates to a logarithm.

(21) **A.** At a vertical asymptote the denominator is zero, so $a+9=0$. Therefore $a=-9$. The numerator is non-zero at $x=3$, so this denominator zero is not cancelled.

(22) **B.** If X counts the selected result and W counts all other results, then $W=20-X$. Hence $q(w)=\Pr(W=w)=\Pr(X=20-w)=p(20-w)$.

(23) **A.** $h'(x)=2\cos(2x)f'(\sin(2x))$. At $x=\frac{\pi}{8}$, both the sine and cosine values are $\frac{\sqrt{2}}{2}$, giving $4\sqrt{2}$.

(24) **D.** Substitution of the point gives $-3=\dfrac{a}{5}$. Therefore $a=5\times-3=-15$.

2. (a) Using the required working, the answer is

$$\int_0^5 kx^2(5-x)\,dx=1,\ k\left[\frac{5x^3}{3}-\frac{x^4}{4}\right]_0^5=k\times\frac{625}{12}=1,\ k=\frac{12}{625}$$

.

(b) Using the required working, the answer is $\Pr(X<3)=\int_0^3\frac{12}{625}x^2(5-x)\,dx,\ \approx 0.4752$.

(c) Using the required working, the answer is $E(X)=\int_0^5 x\times\frac{12}{625}x^2(5-x)\,dx,\ E(X)=3$ hours.

(d) Using the required working, the answer is $E\left(X^2\right)=\int_0^5 x^2\times\frac{12}{625}x^2(5-x)\,dx=10$, $\operatorname{sd}(X)=\sqrt{E\left(X^2\right)-[E(X)]^2}=1,\ \approx 1.00$ hours.

(e) Using the required working, the answer is $\int_0^m\frac{12}{625}x^2(5-x)\,dx=0.5,\ m\approx 3.07$ hours.

3. (a) Using the required working, the answer is $c(0)=0$ milligrams per litre.

(b) Using the required working, the answer is $c'(t)=5e^{-0.4t}(1-0.4t)$, $c'(t)=0$ when $t=\frac{1}{0.4}$, $t\approx 2.50$ hours, $c(2.50)\approx 4.60$ milligrams per litre.

(c) Using the required working, the answer is average$=\frac{1}{8}\int_0^8 5te^{-0.4t}\,dt,\ \approx 3.24$ milligrams per litre.

(d) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives $c'(3) \approx -0.30$ milligrams per litre per hour.

(e) Using the required working, the answer is $5te^{-0.4t} = 3,\ t \approx 5.57$ hours.

(f) Using the required working, the answer is $e^{-0.4t}$ decays faster than t grows, $c(t) \to 0$ milligrams per litre.

4. (a) Set the model equal to the required value and solve for the variable. This gives $d(x) = 0$ when $x = \pm 1$, width$= 2 \approx 2.00$ m.

(b) Using the required working, the answer is $d(0) = 1$ m.

(c) Using the required working, the answer is area$= \int_{-1.00}^{1.00} \left(1 - x^2\right) dx,\ \approx 1.33$ m^2.

(d) Using the required working, the answer is average depth$= \frac{\text{area}}{\text{width}},\ \approx 0.67$ m.

(e) Using the required working, the answer is volume=area$\times 40 \approx 53.33$ m^3.

5. (a) (i) Using the required working, the answer is $r(0) = 2800$.

(ii) Using the required working, the answer is minimum$= 900$, maximum$= 4700$.

(iii) Using the required working, the answer is period$= \frac{2\pi}{\frac{\pi}{70}} = 140$ weeks.

(b) (i) $a = \frac{2300-700}{2} = 800$, The maximum is half a period after the minimum, so the period is $2(100-30) = 140,\ b = \frac{2\pi}{140} = \frac{\pi}{70}$

(ii) Using the required working, the answer is Both populations have period 140, so the sum is a single sinusoid, amplitude$= 1134.12$, maximum$= 4300 + 1134.12 \approx 5434$.

(iii) The sum has the same period as each population, 140 weeks

(c) Using the required working, the answer is Under Q the image of $y = \sin(t)$ is $y = 800\sin\left(\frac{\pi(t-65)}{90}\right) + 1500$, average

$$= \frac{1}{300}\int_0^{300} \left(1900\sin\left(\frac{\pi t}{70}\right) + 2800 + 800\sin\left(\frac{\pi(t-65)}{90}\right) + 1500\right) dt,\ \approx 4330$$

.

(d) (i) Use the average-rate formula $\frac{f(b)-f(a)}{b-a}$ over the given interval. This gives $s'(t) = 0$ where

$$\tan\left(\frac{\pi}{70}t\right) = \frac{\frac{\pi}{70}}{0.004},\ t_1 \approx 33.02, \quad t_2 \approx 173.02$$

, average rate

$$= \frac{s(t_2) - s(t_1)}{t_2 - t_1} \approx -5.1 \text{ per week}$$

.

(ii) $s''(t) = 0$ where

$$\tan\left(\frac{\pi}{70}t\right) = \frac{2 \times 0.004 \times \frac{\pi}{70}}{(0.004)^2 - \left(\frac{\pi}{70}\right)^2}$$

, The decay makes each later positive peak smaller, so the first one after $t = 50$ is the greatest, $t \approx 136$ weeks

(iii) Using the required working, the answer is $e^{-0.004t} \to 0$ as $t \to \infty,\ s(t) \to 2800$.

6. (a) Using the required working, the answer is $\Pr(X > 515) = 0.0668$.

(b) Using the required working, the answer is

$$\Pr(X > 515 \mid X > 495) = \frac{\Pr(X > 515)}{\Pr(X > 495)},\ = \frac{0.0668}{0.6915} = 0.0966$$

.

(c) Using the required working, the answer is $\Pr(X < m) = 0.90,\ m = 512.8$ grams.

(d) Using the required working, the answer is Discrete, $\hat{P} = \frac{X}{200}$, where X counts the underweight apples, so $\hat{P}$ takes only the values $0, \frac{1}{200}, \ldots, 1$.

(e) Using the required working, the answer is

$$\hat{p} = \frac{16}{200} = 0.080,\ \hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{200}},\ (0.042, 0.118)$$

.

(f) The width is proportional to $\frac{1}{\sqrt{n}}$, so halving it needs four times the sample, $n = 4 \times 200 = 800$

(g) Using the required working, the answer is
$X \sim \mathrm{Bi}(8, 0.080)$, $\Pr(X = 2) = 0.1087$, $\Pr(X \geq 2) = 0.1298$.