SUPERVISOR TO ATTACH PROCESSING LABEL HERE

VCE Mathematical Methods
Units 3 & 4
Practice examination

STUDENT NUMBER									Letter

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
7	7	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(4 marks)

a Differentiate the following expression with respect to b

$$-3\sin^2(2b) - 4\tan(d)$$

A

$$-6\sin(4b)$$

B

$$-30\sin(4b)$$

C

$$-\sin^2\left(2d^2\right)$$

D

$$-\frac{3\sin(4b)}{5}$$

(1 mark)

b Integrate the expression with respect to z

$$-\frac{12}{2z+3}$$

A

$$-12\log(2z+3)+C$$

B

$$-6\log(2z+3)+C$$

C

$$-6\log(2z+3)-2+C$$

D

$$-\frac{3\log(2z+3)}{2}+C$$

(1 mark)

c Differentiate the following expression with respect to p:

$$\left(-5p^2-4p+1\right)^{\frac{8}{3}}$$

A

$$-4p^2+2p-5$$

B

$$\frac{8\left(-5p-2\right)\left(-5p^{2}-4p+1\right)^{\frac{5}{3}}}{27}$$

C

$$\frac{224\left(-5p-2\right)\left(-5p^{2}-4p+1\right)^{\frac{5}{3}}}{3}$$

D

$$\frac{16\left(-5p-2\right)\left(-5p^{2}-4p+1\right)^{\frac{5}{3}}}{3}$$

(2 marks)

Question 2

Consider the equation

$$\cos^{2}(\theta)+3\sin(\theta)-3=0,$$

where $\theta \in [0, 2\pi]$. Give all answers in exact form.

a Use the identity $\cos^2(\theta) = 1 - \sin^2(\theta)$ to write the equation as a quadratic in $\sin(\theta)$, and hence state all possible values of $\sin(\theta)$. (2 marks)

b Hence find all solutions of the equation. (2 marks)

Question 3

Consider the function $f(x) = x^2 + 4x + 11$ and the point $P(1, 0)$.

a Show that the point P is not on the graph of $y = f(x)$. (1 mark) b Consider a point $Q(a, f(a))$ on the graph of f. Find the slope of the line connecting P and Q in terms of a. (1 mark)

c Find the slope of the tangent to the graph of f at Q in terms of a. (1 mark)

d Let the tangent to the graph of f at $x = a$ pass through P. Find the values of a. (2 marks)

e Give the equation of one of the lines passing through P that is tangent to the graph of f. (1 mark)

Question 4

On any given day the depth of water alongside a jetty is modelled by the function

$$h(t) = 4\sin\left(\frac{\pi t}{6}\right) + 8, \quad 0 \leq t \leq 12,$$

where h is the depth in metres and t is the time in hours after 8 am. Give all answers in exact form.

a Find the minimum depth. (1 mark)

b Find the values of t for which $h(t) = 10$. (2 marks)

c Find the maximum depth and the value of t at which it occurs. (2 marks)

Question 5

A council reviews each street tree at its yearly assessment. Let A be the event that a randomly chosen street tree will need pruning, and let B be the event that it will need mulching. It is known that

$$\Pr(A) = \frac{1}{2}, \quad \Pr(B) = \frac{2}{5}, \quad \Pr(A \cap B) = \frac{1}{5}.$$

a State the probability that a randomly chosen street tree will need mulching but will not need pruning. (1 mark)

b Find $\Pr(A \cup B)$. (1 mark)

c Find $\Pr(A \mid B)$. (2 marks)

d State, with justification, whether A and B are independent. (2 marks)

Question 6

Consider the system of equations

$$\begin{cases}(k+4)x+4y=2,\\ 3x+ky=k+3\end{cases}$$

where $k \in R$. Give all answers in exact form.

a Find the values of k for which the system has a unique solution. (2 marks)

b Determine the value of k for which the system has an infinite number of solutions. (3 marks)

c Determine the value of k for which the system has no solution. (2 marks)

Question 7

A tilemaker wants to make square tiles of size 40 cm $\times$ 40 cm. The front surface of each tile is painted in two different colours that meet the following conditions.

- Condition 1: each colour covers half the front surface of a tile.
- Condition 2: the tiles can be lined up in a single horizontal row so that the colours form a continuous pattern.

The corners of each tile are at $(0,0)$, $(40,0)$, $(40,40)$ and $(0,40)$. On a Type A tile the colours are divided by

$$f(x)=10\sin\left(\frac{\pi x}{20}\right)+a, \quad a \in R,$$

and on a Type B tile they are divided by

$$g(x)=-\frac{x^3}{400}+\frac{3x^2}{20}-2x+20.$$

Give all answers in exact form.

a Find the value of a so that a Type A tile meets Condition 1. (2 marks)

b Show that a Type B tile meets Condition 1. (3 marks)

c Determine the endpoints of f and g on each tile, and hence confirm that Type A and Type B tiles can be placed in any order to meet Condition 2. (1 mark)

d Find the gradient of the Type B dividing curve at each vertical edge of a tile, and state what this means when two Type B tiles are placed side by side. (2 marks)

Solutions

1. (a) A
   (b) B
   (c) D
2. (a)

   $$\left(1-\sin^2(\theta)\right)+3\sin(\theta)-3=0,\ \sin^2(\theta)-3\sin(\theta)+2=0,\ (\sin(\theta)-2)(\sin(\theta)-1)=0,\ \sin(\theta)=1$$

   or $\sin(\theta)=2$, Since $-1\leq\sin(\theta)\leq 1$, the value 2 is rejected

   (b) Using the required working, the answer is $\sin(\theta)=1,\ \theta=\frac{\pi}{2}$.
3. (a) Using the required working, the answer is $f(1)=16$, which is not 0, so P is not on the graph..
   (b) Using the required working, the answer is $m_{PQ}=\frac{f(a)-0}{a-1}=\frac{a^2+4a+11}{a-1}$.
   (c) Using the required working, the answer is $f'(a)=2a+4$.
   (d) Using the required working, the answer is Equating the two slopes: $2a+4=\frac{a^2+4a+11}{a-1},\ a^2-2a-15=0,\ a=-3,\ 5$.

(e) Using the required working, the answer is At $a=5,\ f'(a)=14,\ y=14x-14$.

4. (a) The sine term has minimum value $-1,\ h_{\min}=8-4=4$ metres
   (b) Using the required working, the answer is $\sin\left(\frac{2\pi t}{12}\right)=\frac{1}{2},\ t=1,\ 5$.
   (c) Using the required working, the answer is $h_{\max}=8+4=12$ metres, $t=3$ hours after 8 am.

5. (a) Using the required working, the answer is $\Pr(B\cap A')=\Pr(B)-\Pr(A\cap B)=\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$.
   (b) Using the required working, the answer is

$$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)=\frac{1}{2}+\frac{2}{5}-\frac{1}{5},=\frac{7}{10}$$

.

   (c) Using the required working, the answer is $\Pr(A\mid B)=\frac{\Pr(A\cap B)}{\Pr(B)}=\frac{\frac{1}{5}}{\frac{2}{5}},=\frac{1}{2}$.
   (d) $\Pr(A)\Pr(B)=\frac{1}{2}\times\frac{2}{5}=\frac{1}{5},\ \Pr(A\cap B)=\frac{1}{5}$, Since $\Pr(A\cap B)=\Pr(A)\Pr(B)$, the events are independent.

6. (a) Using the required working, the answer is A unique solution requires the determinant of the coefficient matrix to be non-zero

$$,\ k^2+4k-12\neq 0,\ (k-2)(k+6)\neq 0,\ k\in R\setminus\{-6,2\}$$

.

   (b) The determinant is zero when $k=-6$ or $k=2$, At $k=-6$ the second equation is a multiple of the first, so the system is consistent, $k=-6$
   (c) Using the required working, the answer is At $k=2$ the left sides are proportional but the right sides are not, so the equations are inconsistent, $k=2$.

7. (a) Using the required working, the answer is Half the tile area is

$$\frac{1600}{2}=800,\ \int_0^{40}\left(10\sin\left(\frac{\pi x}{20}\right)+a\right)dx=40a,\ 40a=800,\ a=20$$

.

   (b) Using the required working, the answer is

$$\int_0^{40}\left(-\frac{x^3}{400}+\frac{3x^2}{20}-2x+20\right)dx=\left[-\frac{x^4}{1600}+\frac{x^3}{20}-x^2+20x\right]_0^{40},=800$$

, which is half of 1600, so each colour covers half the tile.

   (c) Using the required working, the answer is $f(0)=f(40)=20,\ g(0)=g(40)=20$, so every tile meets its vertical edges at height 20 and the colours join in any order.
   (d) Using the required working, the answer is $g'(x)=-\frac{3x^2}{400}+\frac{3x}{10}-2,\ g'(0)=g'(40)=-2$, so the curve leaves one tile at the same gradient as it enters the next and the pattern joins smoothly.