SUPERVISOR TO ATTACH PROCESSING LABEL HERE

VCE Mathematical Methods
Units 3 & 4
Practice examination

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(3 marks)

a Differentiate the following expression with respect to q:

$$\left(5q^2r^2+3qr+1\right)^6$$

A

$$-q^2-r+1$$

B

$$108r\,(10qr+3)\left(5q^2r^2+3qr+1\right)^5$$

C

$$6r\,(10qr+3)\left(5q^2r^2+3qr+1\right)^5$$

D

$$\frac{6r\,(10qr+3)\left(5q^2r^2+3qr+1\right)^5}{5}$$

(2 marks)

b Evaluate the following indefinite integral

$$\int -\frac{6}{(v+1)\,(5v+2)}\,dv$$

. Hint: partition the fraction into simpler fractions before integrating.

A

$$\log(3v+3)-\log(5v+2)+C$$

B

$$10\log(3v+3)-10\log(5v+2)+C$$

C

$$-\frac{6}{(v+1)\,(5v+2)}+C$$

D

$$2\log(3v+3)-2\log(5v+2)+C$$

(1 mark)

Question 2

Part of the graph of $y = f(x)$ lies above the horizontal axis. The rule

$$A(k) = 2k\sin(k)$$

gives the area bounded by the graph of f, the horizontal axis and the line $x = k$, for $k > 0$. Give all answers in exact form.

a State the value of $A\left(\frac{\pi}{6}\right)$. (1 mark)

b Explain why $f(k) = A'(k)$, and hence evaluate $f\left(\frac{\pi}{6}\right)$. (2 marks)

c Find the area bounded by the graph of f, the horizontal axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$. (2 marks)

Question 3

Let $(\tan(\theta) - 1)\left(\sin(\theta) - \sqrt{3}\cos(\theta)\right)\left(\sin(\theta) + \sqrt{3}\cos(\theta)\right) = 0$. Give all answers in exact form.

a State all possible values of $\tan(\theta)$. (1 mark)

b Hence find all possible solutions for $(\tan(\theta) - 1)\left(\sin^2(\theta) - 3\cos^2(\theta)\right) = 0$, where $0 \leq \theta \leq \pi$. (2 marks)

Question 4

Consider the function $f(x) = x^2 + 3x - 6$ and the point $P(2,0)$.

a Show that the point P is not on the graph of $y = f(x)$. (1 mark)

b Consider a point $Q(a, f(a))$ on the graph of f. Find the slope of the line connecting P and Q in terms of a. (1 mark)

c Find the slope of the tangent to the graph of f at Q in terms of a. (1 mark)

d Let the tangent to the graph of f at $x = a$ pass through P. Find the values of a. (2 marks)

e Give the equation of one of the lines passing through P that is tangent to the graph of f. (1 mark)

Question 5

Consider the equation

$$2\cos^2(\theta) - \sin(\theta) + 1 = 0,$$

where $\theta \in [0, 2\pi]$. Give all answers in exact form.

a Use the identity $\cos^2(\theta) = 1 - \sin^2(\theta)$ to write the equation as a quadratic in $\sin(\theta)$, and hence state all possible values of $\sin(\theta)$. (2 marks)

b Hence find all solutions of the equation. (2 marks)

Question 6

Let $f : D \to R,\ f(x) = 2\log_4(x+2) - \log_4(x+5)$, where D is the maximal domain of f. Give all answers in exact form.

a State the maximal domain D of f. (2 marks)

b Show that $f(x) = \log_4\left(\frac{(x+2)^2}{x+5}\right)$ for $x \in D$. (1 mark)

c Hence solve $f(x) = 1$. (3 marks)

Question 7

A council reviews each street tree at its yearly assessment. Let A be the event that a randomly chosen street tree will need pruning, and let B be the event that it will need mulching. It is known that

$$\Pr(A) = \frac{1}{2}, \quad \Pr(B) = \frac{2}{5}, \quad \Pr(A \cap B) = \frac{1}{5}.$$

a State the probability that a randomly chosen street tree will need mulching but will not need pruning. (1 mark)

b Find $\Pr(A \cup B)$. (1 mark)

c Find $\Pr(A \mid B)$. (2 marks)

d State, with justification, whether A and B are independent. (2 marks)

Question 8

Let $f : D \to R,\ f(x) = \log_e\left(x^2 + x - 12\right)$, where D is the maximal domain of f. Give all answers in exact form.

a Find the maximal domain D of f. (3 marks)

b State the equations of the vertical asymptotes of the graph of f. (1 mark) c Solve $f(x)=\log_e(18)$ for x. (3 marks)

Solutions

1. (a) C
   (b) D
2. (a) Using the required working, the answer is $A\left(\frac{\pi}{6}\right)=\frac{\pi}{6}$.
   (b) Using the required working, the answer is By the fundamental theorem of calculus, differentiating the area function returns the integrand, so

$$f(k)=A'(k),\ f(k)=2k\cos(k)+2\sin(k),\ f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}\pi}{6}+1$$

.

   (c) Using the required working, the answer is

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} f(x)\,dx=A\left(\frac{\pi}{3}\right)-A\left(\frac{\pi}{6}\right),=\frac{\pi\left(-1+2\sqrt{3}\right)}{6}$$

.

3. (a) Using the required working, the answer is $\tan(\theta)=1$ or $\tan(\theta)=\pm\sqrt{3},\ \tan(\theta)=-\sqrt{3},\ 1,\ \sqrt{3}$.
   (b) Using the required working, the answer is

$$\sin^2(\theta)-3\cos^2(\theta)=\left(\sin(\theta)-\sqrt{3}\cos(\theta)\right)\left(\sin(\theta)+\sqrt{3}\cos(\theta)\right)$$

, so the equation has the same solutions as part a, $\theta=\frac{\pi}{4},\ \frac{\pi}{3},\ \frac{2\pi}{3}$.

4. (a) Using the required working, the answer is $f(2)=4$, which is not 0, so P is not on the graph..
   (b) Using the required working, the answer is $m_{PQ}=\frac{f(a)-0}{a-2}=\frac{a^2+3a-6}{a-2}$.
   (c) Using the required working, the answer is $f'(a)=2a+3$.
   (d) Using the required working, the answer is Equating the two slopes: $2a+3=\frac{a^2+3a-6}{a-2},\ a(a-4)=0,\ a=0,\ 4$.
   (e) Using the required working, the answer is At $a=4,\ f'(a)=11,\ y=11x-22$.
5. (a)

$$2\left(1-\sin^2(\theta)\right)-\sin(\theta)+1=0,\ 2\sin^2(\theta)+\sin(\theta)-3=0,\ (\sin(\theta)-1)(2\sin(\theta)+3)=0,\ \sin(\theta)=1$$

or $\sin(\theta)=-\frac{3}{2}$, Since $-1\leq\sin(\theta)\leq 1$, the value $-\frac{3}{2}$ is rejected

   (b) Using the required working, the answer is $\sin(\theta)=1,\ \theta=\frac{\pi}{2}$.
6. (a) Using the required working, the answer is Both logarithms require a positive argument: $x+2>0$ and $x+5>0,\ x>-2$ and $x>-5,\ D=(-2,\infty)$.

(b) Using the required working, the answer is

$$2\log_4(x+2)=\log_4\left((x+2)^2\right),\ \log_4\left((x+2)^2\right)-\log_4(x+5)=\log_4\left(\frac{(x+2)^2}{x+5}\right)$$

.

(c) Using the required working, the answer is

$$\log_4\left(\frac{(x+2)^2}{x+5}\right)=1,\ (x+2)^2=4(x+5),\ x^2-16=0,\ x=-4,\ 4$$

, From part a, $D=(-2,\infty)$, so $x=-4$ is rejected and $x=4$.

7. (a) Using the required working, the answer is $\Pr(B\cap A')=\Pr(B)-\Pr(A\cap B)=\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$.

(b) Using the required working, the answer is

$$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)=\frac{1}{2}+\frac{2}{5}-\frac{1}{5},\ =\frac{7}{10}$$

.

(c) Using the required working, the answer is $\Pr(A\mid B)=\frac{\Pr(A\cap B)}{\Pr(B)}=\frac{\frac{1}{5}}{\frac{2}{5}},\ =\frac{1}{2}$.

(d) $\Pr(A)\Pr(B)=\frac{1}{2}\times\frac{2}{5}=\frac{1}{5},\ \Pr(A\cap B)=\frac{1}{5}$, Since $\Pr(A\cap B)=\Pr(A)\Pr(B)$, the events are independent.

8. (a) The logarithm requires a positive argument:

$$x^2+x-12>0,\ (x-3)(x+4)>0,\ D=(-\infty,-4)\cup(3,\infty)$$

(b) The argument approaches 0 at the ends of D, where $(x-3)(x+4)=0,\ x=-4,\quad x=3$

(c) Using the required working, the answer is
$x^2+x-12=18,\ x^2+x-30=0,\ (x-5)(x+6)=0,\ x=-6,\ 5$, Both solutions lie in D, so both are accepted..