SUPERVISOR TO ATTACH PROCESSING LABEL HERE

VCE Mathematical Methods
Units 3 & 4
Practice examination

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(3 marks)

a Differentiate the following expression with respect to n

$$\frac{\sqrt{2}\,(n+4)}{2\sqrt{2n^2-n-2}}$$

A

$$3n^3+n^2-4n+1$$

B

$$-\frac{17\sqrt{2}n}{4\,(2n^2-n-2)^{\frac{3}{2}}}$$

C

$$-\frac{255\sqrt{2}n}{4\,(2n^2-n-2)^{\frac{3}{2}}}$$

D

$$-\frac{17\sqrt{2}n}{20\,(2n^2-n-2)^{\frac{3}{2}}}$$

(1 mark)

b Differentiate the polynomial with respect to c

$$\frac{1-4c}{c^2}$$

A

$$\frac{2\,(2c-1)}{c^3}$$

B

$$4-5c$$

C

$$\frac{10\,(2c-1)}{c^3}$$

D

$$\frac{2c-1}{5c^3}$$

(1 mark)

c Differentiate $f(n)$.

$$f(n) = -\cos(2n)$$

Hence, find the following integral.

$$\int 2\sin(2n) + \frac{1}{n}\, dn$$

A

$$\log(n) - \cos(2n) + C$$

B

$$-\log(n) - \cos(2n) + C$$

C

$$-\cos(2n) + C$$

D

$$2\log(n) - \cos(2n) + C$$

(1 mark)

Question 2

Let $g : \left(\frac{3}{2}, \infty\right) \to R,\ g(x) = \frac{3}{2x-3}$, and let f be a function for which

$$\int_0^1 f(x)\, dx = \frac{1}{3} \quad \text{and} \quad \int_0^1 [f(x)]^2\, dx = \frac{1}{5}.$$

a Find the rule for an antiderivative of $g(x)$. (1 mark)

b Evaluate $\int_0^1 f(x)\,(3f(x)+2)\, dx$. (3 marks)

c Evaluate $\int_2^3 g(x)\, dx$. (2 marks)

Question 3

Let $\left(\tan(\theta) - \frac{\sqrt{3}}{3}\right)\left(\sin(\theta) - \sqrt{3}\cos(\theta)\right)\left(\sin(\theta) + \sqrt{3}\cos(\theta)\right) = 0$. Give all answers in exact form.

a State all possible values of $\tan(\theta)$. (1 mark)

b Hence find all possible solutions for $\left(\tan(\theta) - \frac{\sqrt{3}}{3}\right)\left(\sin^2(\theta) - 3\cos^2(\theta)\right) = 0$, where $0 \leq \theta \leq \pi$. (2 marks)

Question 4

Let $y = 4(x+2)e^{3x}$ and let $f : R \to R,\ f(x) = \frac{\sin(x)}{e^{2x}}$. Give all rules in exact form.

a Find $\dfrac{dy}{dx}$. (1 mark)

b Find and simplify the rule of $f'(x)$. (2 marks)

c Find the equation of the tangent to the graph of $y = 4(x+2)e^{3x}$ at the point where $x = 0$. (2 marks)

Question 5

Consider the equation

$$\sin(2\theta) - \sin(\theta) = 0.$$

Give all answers in exact form.

a Use the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ to factorise the equation, and hence state all possible values of $\sin(\theta)$ and $\cos(\theta)$. (1 mark)

b Hence find all solutions for $\theta \in [0, 2\pi]$. (2 marks)

c State the number of solutions for $\theta \in [0, 4\pi]$. (1 mark)

Question 6

Let $f : R \to R,\ f(x) = 2^{4x-11}$. Give all answers in exact form.

a Solve $f(x) = 8$ for x. (2 marks)

b Find the value of x for which $f(x) = 1$. (1 mark) c State the range of f. (1 mark)

d Find the rule and the domain of f^{-1}, the inverse function of f. (3 marks)

Question 7

Two tins contain buttons that differ only in colour. Tin 1 contains one silver button and four gold buttons. Tin 2 contains five gold buttons. A tin is chosen randomly and one button is drawn randomly from it. Each tin is equally likely to be chosen, as is each button. Let T_i be the event that Tin i is chosen, and let A be the event that the button drawn is gold.

a Find $\Pr(T_2 \cap A)$. (1 mark)

b Find $\Pr(A)$. (2 marks)

c Hence find $\Pr(T_2 \mid A)$. (2 marks)

Question 8

Let $f : D \to R,\ f(x) = \log_e\left(x^2 - 2x - 3\right)$, where D is the maximal domain of f. Give all answers in exact form.

a Find the maximal domain D of f. (3 marks)

b State the equations of the vertical asymptotes of the graph of f. (1 mark)

c Solve $f(x) = \log_e(12)$ for x. (3 marks)

Solutions

1. (a) B
   (b) A
   (c) A
2. (a) Integrate the given expression and simplify. This gives $\int \frac{3}{2x-3}\,dx = \frac{3\log_e(2x-3)}{2} + C$.
   (b) Using the required working, the answer is
   $$\int_0^1 f(x)\,(3f(x)+2)\,dx = 3\int_0^1 [f(x)]^2\,dx + 2\int_0^1 f(x)\,dx,\ = \frac{19}{15}$$
   .
   (c) Using the required working, the answer is $\int_2^3 g(x)\,dx = \frac{3\log_e(3)}{2}$.
3. (a) Using the required working, the answer is
   $\tan(\theta) = \frac{\sqrt{3}}{3}$ or $\tan(\theta) = \pm\sqrt{3},\ \tan(\theta) = -\sqrt{3},\ \frac{\sqrt{3}}{3},\ \sqrt{3}$.
   (b) Using the required working, the answer is
   $$\sin^2(\theta) - 3\cos^2(\theta) = \left(\sin(\theta) - \sqrt{3}\cos(\theta)\right)\left(\sin(\theta) + \sqrt{3}\cos(\theta)\right)$$
   , so the equation has the same solutions as part a, $\theta = \frac{\pi}{6},\ \frac{\pi}{3},\ \frac{2\pi}{3}$.
4. (a) Using the required working, the answer is $\frac{dy}{dx} = (12x+28)\,e^{3x}$.
   (b) Using the required working, the answer is $f'(x) = (-2\sin(x) + \cos(x))\,e^{-2x}$.
   (c) Using the required working, the answer is $\left.\frac{dy}{dx}\right|_{x=0} = 28,\ y = 28x + 8$.
5. (a) Using the required working, the answer is
   $\sin(\theta)\,(2\cos(\theta) - 1) = 0,\ \sin(\theta) = 0$ or $\cos(\theta) = \frac{1}{2}$.

(b) Using the required working, the answer is

$$\sin(\theta)=0 \quad \Rightarrow \quad \theta=0,\ \pi,\ 2\pi,\ \cos(\theta)=\frac{1}{2} \quad \Rightarrow \quad \theta=\frac{\pi}{3},\ \frac{5\pi}{3},\ \theta=0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 2\pi$$

.

(c) The equation repeats every 2π, and a solution at an endpoint is not counted twice

$$,\ \theta=0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 2\pi,\ \frac{7\pi}{3},\ 3\pi,\ \frac{11\pi}{3},\ 4\pi$$

, number of solutions$=9$

6. (a) Using the required working, the answer is $8=2^3,\ 4x-11=3,\ x=\frac{7}{2}$.
   (b) Using the required working, the answer is $4x-11=0,\ x=\frac{11}{4}$.
   (c) Determine the possible output values from the function. The range is range$(f)=(0,\infty)$.
   (d) Write $y=f(x)$, swap x and y, and solve for y. This gives

$$y=2^{4x-11},\ \log_2(y)=4x-11,\ x=\frac{\log_2(y)+11}{4},\ f^{-1}(x)=\frac{\log_2(x)+11}{4},$$

domain$=(0,\infty)$.

7. (a) Using the required working, the answer is

$$\Pr(A\mid T_2)=1,\ \Pr(T_2\cap A)=\Pr(T_2)\Pr(A\mid T_2)=\frac{1}{2}\times 1=\frac{1}{2}$$

.

(b) Using the required working, the answer is

$$\Pr(A)=\Pr(T_1\cap A)+\Pr(T_2\cap A),\ =\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times 1,\ =\frac{9}{10}$$

.

(c) Using the required working, the answer is

$$\Pr(T_2\mid A)=\frac{\Pr(T_2\cap A)}{\Pr(A)},\ =\frac{\frac{1}{2}}{\frac{9}{10}},\ =\frac{5}{9}$$

.

8. (a) The logarithm requires a positive argument:

$$x^2-2x-3>0,\ (x-3)(x+1)>0,\ D=(-\infty,-1)\cup(3,\infty)$$

   (b) The argument approaches 0 at the ends of D, where $(x-3)(x+1)=0,\ x=-1, \quad x=3$
   (c) Using the required working, the answer is

$$x^2-2x-3=12,\ x^2-2x-15=0,\ (x-5)(x+3)=0,\ x=-3,\ 5$$

, Both solutions lie in D, so both are accepted..