MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	**9**	**49**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(13 marks)

a If

$$f(x) = -6 + \frac{1}{x+1}$$

and

$$g(x) = -3x - 3$$

let

$$h(x) = f(g(x))$$

. What is the domain of $h(x)$?

A

$$(-\infty, -6) \cup (-6, \infty)$$

B

$$\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \infty\right)$$

C

$$\mathbb{R}$$

D

$$\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$$

(1 mark)

b Solve the following equation for x:

$$|5x - 5| = 25$$

A -5 and 7

B -4 and 6

C -4 and 7

D -5 and 6

(1 mark)

c If

$$f(x) = 2 + \frac{1}{x+5}$$

and

$$g(x) = 3x - 4$$

let

$$h(x) = f(g(x))$$

What are the domain and range of $h(x)$?

A

Domain: $(-\infty, 2) \cup (2, \infty)$, Range: $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

B

Domain: $\mathbb{R}$, Range: $(-\infty, 2) \cup (2, \infty)$

C

Domain: $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$, Range: $(-\infty, 2) \cup (2, \infty)$

D

Domain: $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$, Range: $(-\infty, 2) \cup (2, \infty)$

(1 mark)

d If $f(x)$ is defined as

$$f(x) = -x - 4$$

and $g(x)$ is defined as

$$g(x) = 4x^2 - 3x - 4$$

simplify $g(f(x))$.

A

$$4x^2 - 4x - 8$$

B

$$4x^2 - 2x$$

C

$$8x^2 - 6x - 8$$

D

$$4x^2 + 35x + 72$$

(1 mark)

e Sketch the graph of the function

$$f(x) = |(1-x)(-x-1)(x-3)(x+3)|$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = 4x + 4$$

and $g(x)$ is defined as

$$g(x) = x - 5$$

solve the equation

$$g(f(x)) + f(g(x)) = 0$$

A

$$\frac{26}{79}$$

B

$$\frac{17}{8}$$

C

$$\frac{17}{50}$$

D

$$\frac{8}{17}$$

(1 mark)

g If $f(x)$ is defined as

$$f(x) = -1 + \frac{1}{6-x}$$

what is the maximal domain of $f^{-1}(x)$?

A

$$(-\infty, 6) \cup (6, \infty)$$

B

$$(-\infty, -1) \cup (-1, \infty)$$

C

$$\mathbb{R}$$

D

$$(-1, \infty)$$

(1 mark)

h If $f(x)$ is defined as

$$f(x) = \sqrt{4x-1}$$

find $f^{-1}(x)$.

A

$$\left[\frac{x^2}{4} + \frac{1}{4}\right]$$

B

$$\left[\frac{x^2}{1444} + \frac{1}{4}\right]$$

C

$$\left[\frac{361x^2}{4} + \frac{1}{4}\right]$$

D

$$\left[\frac{(x-20)^2}{4} + \frac{1}{4}\right]$$

(1 mark)

i If $f(x)$ is defined as

$$f(x) = \sqrt[3]{4-2x}$$

find $f^{-1}(x)$.

A

$$\left[\frac{(x-8)^3}{2} + 2\right]$$

B

$$\left[2 - 256x^3\right]$$

C

$$\left[2 - \frac{x^3}{2}\right]$$

D

$$\left[2 - \frac{x^3}{54}\right]$$

(1 mark)

j If $f(x)$ is defined as

$$f(x) = 2\sqrt{x}$$

what is the maximal range of $f^{-1}(x)$?

A

$$[0,\infty)$$

B

$$(-\infty,0]$$

C

$$\mathbb{R}$$

D

$$(0,\infty)$$

(1 mark)

k A line is parallel to the graph

$$-8x-8y-9=0$$

and passes through the point $(-17,8)$. What is the equation of that line?

A

$$y=-x-9$$

B

$$y=x+11$$

C

$$y=-8x-8y-9$$

D

$$y=-4x-36$$

(1 mark)

l A line is perpendicular to the graph

$$-10x+4y-5=0$$

and passes through the point $(2,5.2)$. What is the equation of that line?

A

$$y=-10x+4y-5$$

B

$$y=36-\frac{12x}{5}$$

C

$$y=\frac{2x}{5}+4$$

D

$$y=6-\frac{2x}{5}$$

(1 mark)

m For the following two functions,

$$f(x)=2ax^2-5ax+1$$

and

$$g(x)=-4ax-2$$

what values of a will ensure that the graphs of these functions never intersect?

A

$a > -19, a < 17$

B

$a \leq 13, a < -18$

C

$a \geq 6, a \leq -11$

D

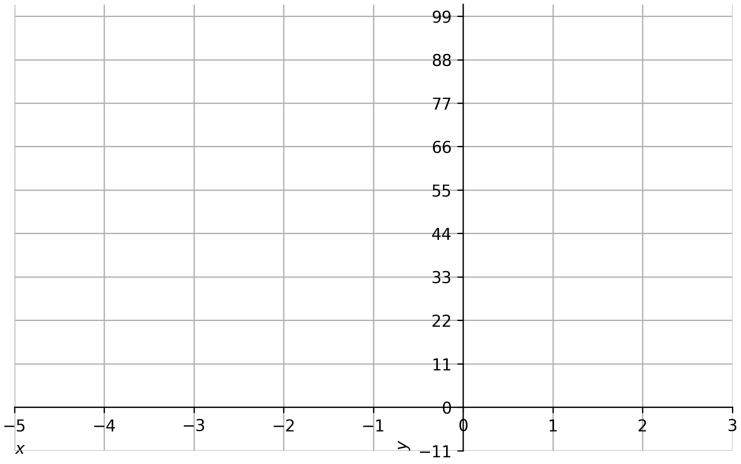
$a > 0, a < 24$

(1 mark)

Question 2

Let $f : R \to R$, $f(x) = x^4 + 3x^3 - 12x^2 - 20x + 48$.

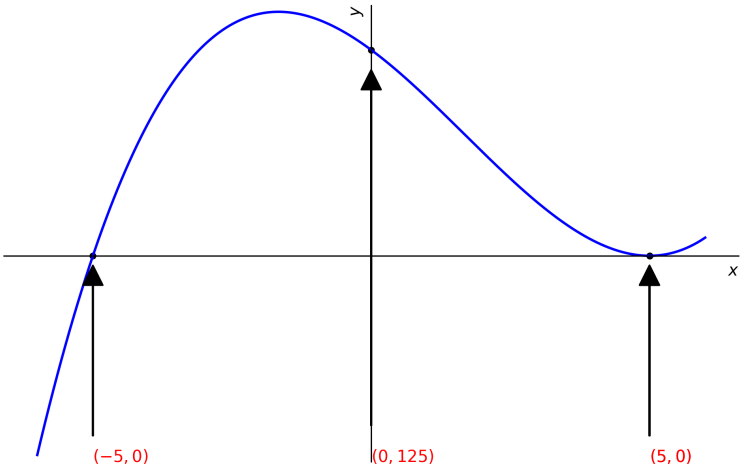
Sketch the graph of $f(x)$ on the following grid.



(4 marks)

Question 3

Consider the following graph.



a Write the rule for the graph. (1 mark)

b This function is transformed according to the rule

$$g(x) = -2f(x-5)$$

Simplify $g(x)$ in terms of x. (2 marks)

c Find the maximal domain of $f(x)$. (1 mark)

d Another function $g(x)$ is defined as

$$g(x) = 3f(x)x^4 - 3x$$

Simplify $g(x)$ in terms of x. (1 mark)

Question 4

Consider the function $f(x) = 25x^2 - 10x - 3$.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

c If another function $g(x)$ is defined as

$$g(x) = 5 - 4x$$

solve the equation

$$g(f(x)) = 0$$

(2 marks)

Question 5

Consider the function $f(x) = -4ax^2 + 2a + 5x$.

a What is the domain of $f(x)$ if $a = 5$? (1 mark)

b What is the range of $f(x)$ if $a = -9$? (1 mark)

c If another function $g(x)$ is defined as

$$g(x) = 5x - 2$$

solve the equation

$$g(f(x)) = 0$$

(2 marks)

d This parabola interset with the line $y = a(2x+1)$. Find the value(s) of x at which the two graphs intersect. (3 marks)

Question 6

Consider the function $f : D \to R$ where the rule is $f(x) = \sqrt{-5x-2}$. D is the maximal domain of the function.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

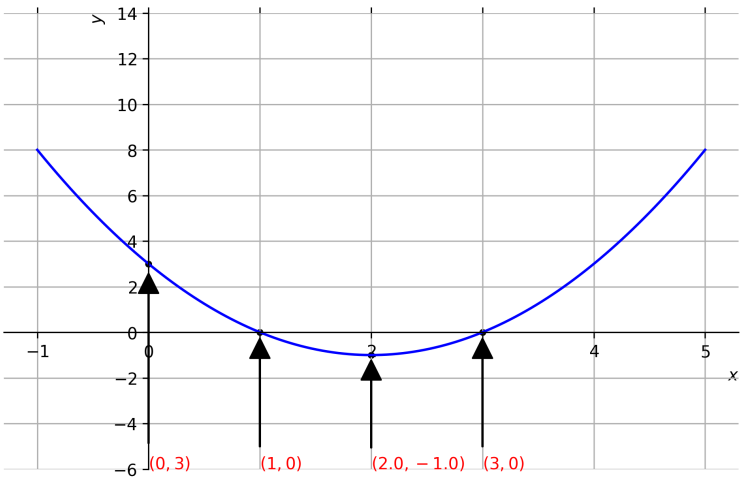
c Find $f^{-1}(x)$. (1 mark)

Question 7

The parabola below is transformed in the following order:

1. horizontal dilation by a factor of -3
2. vertical dilation by a factor of $\frac{1}{-2}$
3. horizontal translation 1 units to the right

Sketch the transformed graph in the following grid.



(3 marks)

Question 8

Consider the function $f : D \to R$ where the rule is $f(x) = -\tan(2x)$.

a State the maximal domain of f. (1 mark)

b State the period of f. (1 mark)

c State the range of f. (1 mark)

d Solve the equation $f(x) = 1$ for $x \in \mathbb{R}$. (3 marks)

Question 9

Consider the functions $f : D \to R$, $f(x) = -12x^2 + 23x - 5$ and $g : D \to R$, $g(x) = e^{-3x}$.

a Find the maximal domain of $f(g(x))$. (1 mark)

b Simplify $g(f(x))$. (1 mark)

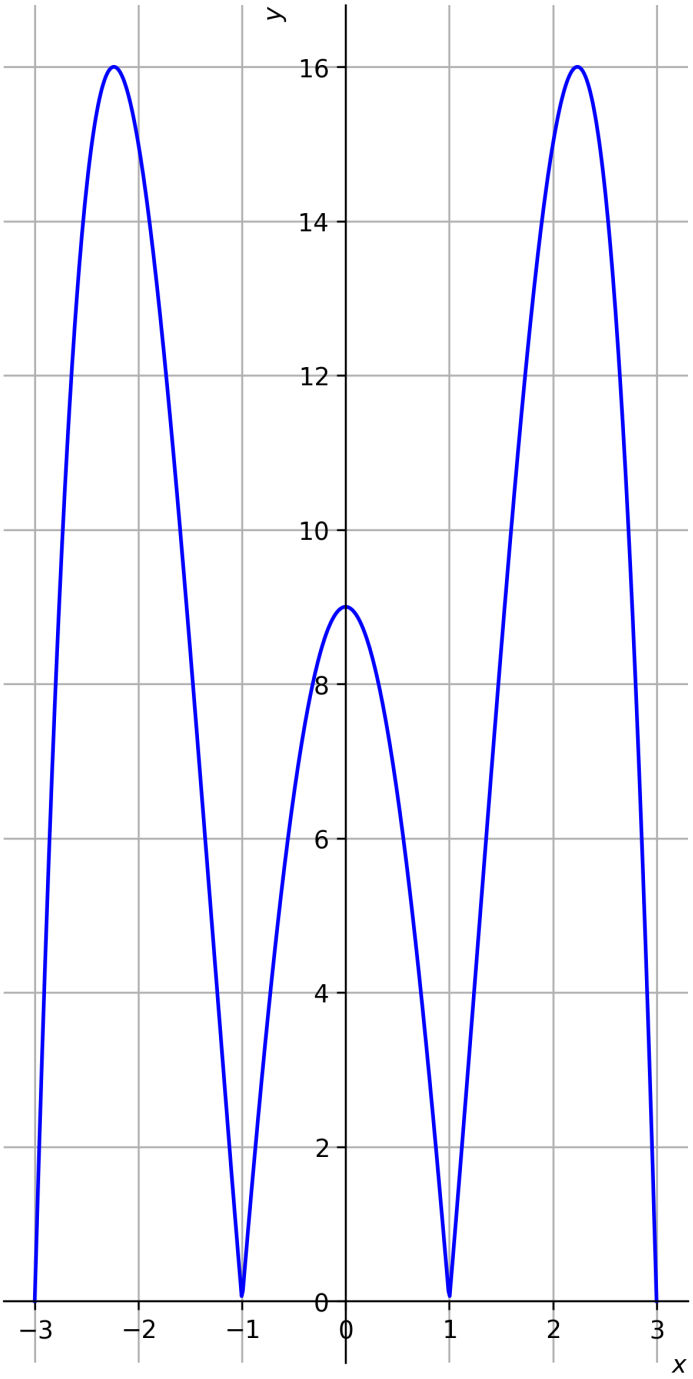
c Solve the equation

$$f(g(x)) = 0$$

(2 marks)

Solutions

1. (a) B
   (b) B
   (c) C
   (d) D



(e)

(f) B

(g) B

(h) A

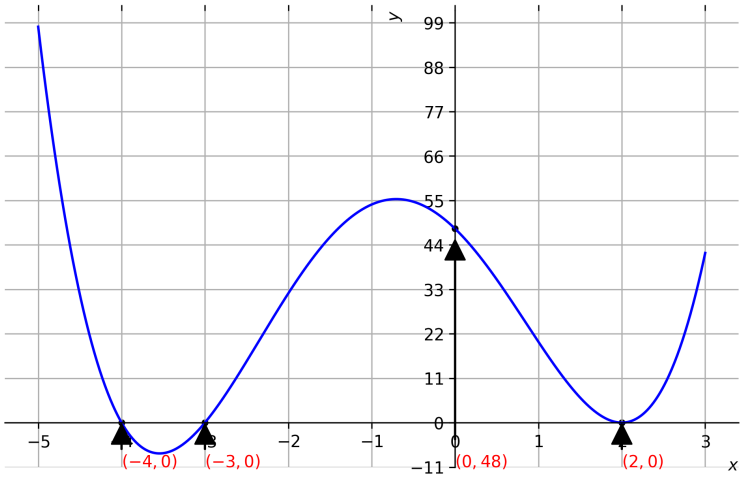
(i) C

(j) A

(k) A

(l) D

(m) D

2. 

3. (a) $x^3 - 5x^2 - 25x + 125$

(b) $-2x(x-10)^2$

(c) $\mathbb{R}$

(d) $3x^7 - 15x^6 - 75x^5 + 375x^4 - 3x$

4. (a) $\mathbb{R}$

(b) $[-4, \infty)$

(c) $\left[\frac{1}{5} - \frac{\sqrt{21}}{10}, \frac{1}{5} + \frac{\sqrt{21}}{10}\right]$

5. (a) $\mathbb{R}$

(b) $\left[-\frac{2617}{144}, \infty\right)$

(c) $\left[\frac{\frac{5}{8} - \frac{\sqrt{800a^2-160a+625}}{40}}{a}, \frac{\frac{\sqrt{800a^2-160a+625}}{40} + \frac{5}{8}}{a}\right]$

(d) $\frac{-\frac{a}{4} - \frac{\sqrt{20a^2-20a+25}}{8} + \frac{5}{8}}{a}$

and

$\frac{-\frac{a}{4} + \frac{\sqrt{20a^2-20a+25}}{8} + \frac{5}{8}}{a}$

6. (a)

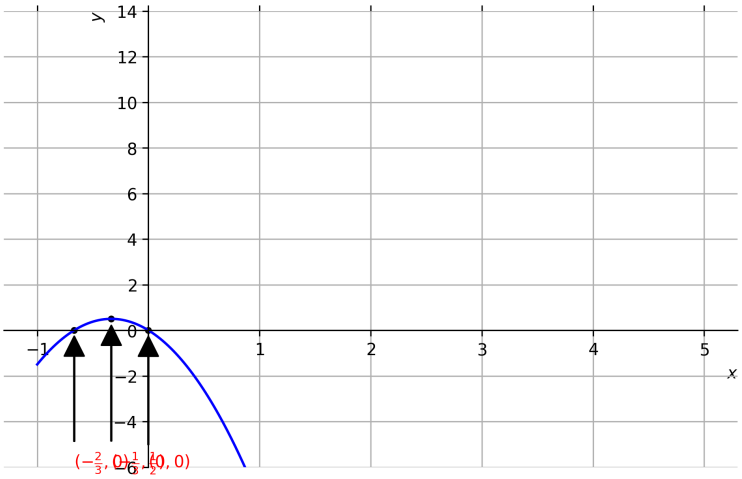
$$\left(-\infty, -\frac{2}{5}\right]$$

(b)

$$[0, \infty)$$

(c)

$$-\frac{x^2}{5} - \frac{2}{5}$$

7. 

8. (a) $\mathbb{R} \setminus \left\{ \frac{\pi(2n+1)}{4} : n \in \mathbb{Z} \right\}$

(b) $\frac{\pi}{2}$

(c) $\mathbb{R}$

(d) $x = \frac{\pi(4n-1)}{8},\ n \in \mathbb{Z}$

9. (a)

$$\mathbb{R}$$

(b)

$$e^{3(3x-5)(4x-1)}$$

(c) There are no real solutions.