MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	**8**	**44**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(15 marks)

a If $f(x)$ is defined as

$$f(x) = 5 - 3x$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{20}{3}\right]$$

B Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{14}{3}\right]$$

C Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{17}{3}\right]$$

D Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{29}{3}\right]$$

(1 mark)

b If $f(x)$ is defined as

$$f(x) = (-2x - 3)(2x - 4)$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{21}{4}\right]$$

B Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{31}{4}\right]$$

C Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{37}{4}\right]$$

D Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{49}{4}\right]$$

(1 mark)

c Does the following sketch represent the graph of the function

$$f(x) = |(-x-4)(x-3)|$$

?

(1 mark)

d Sketch the graph of the function

$$f(x) = x + |x-3|$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = \frac{-x-3}{5-x}$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$(-\infty, 5) \cup (5, \infty)$$

Range:

$$(-\infty, 1) \cup (1, \infty)$$

B Domain:

$$(-\infty, 24) \cup (24, \infty)$$

Range:

$$(-\infty, -8) \cup (-8, \infty)$$

C Domain:

$$(-\infty, 23) \cup (23, \infty)$$

Range:

$$(-\infty, -3) \cup (-3, \infty)$$

D Domain:

$$(-\infty, 23) \cup (23, \infty)$$

Range:

$$(-\infty, -5) \cup (-5, \infty)$$

(1 mark)

f Solve the following equation for x:

$$|x+1| = 3$$

A -4 and 3
B -5 and 3
C -4 and 2
D -5 and 2

(1 mark)

g If $f(x)$ is defined as

$$f(x) = x^2 - 4x + 3$$

and $g(x)$ is defined as

$$g(x) = -4f(-5x^2 + 3x + 5)x^2 - 4x$$

simplify $g(x)$ in terms of x.

A

$$-100x^6 + 120x^5 + 84x^4 - 72x^3 - 32x^2 - 4x$$

B

$$-100x^6 + 120x^5 + 84x^4 - 72x^3 - 31x^2 - 8x + 3$$

C

$$-1100x^6 + 1320x^5 + 924x^4 - 792x^3 - 352x^2 - 44x$$

D

$$-100x^6 + 120x^5 + 84x^4 - 72x^3 - 33x^2 - 3$$

(1 mark)

h If $f(x)$ is defined as

$$f(x) = -4x - 5$$

and $g(x)$ is defined as

$$g(x) = 2x^2 - 2x + 2$$

simplify $g(f(x))$.

A

$$2x^2 - 6x - 3$$

B

$$10x^2 - 10x + 10$$

C

$$32x^2 + 88x + 62$$

D

$$2x^2 + 2x + 7$$

(1 mark)

i If $f(x)$ is defined as

$$f(x) = 2ax + 3x^2$$

and $g(x)$ is defined as

$$g(x) = 3 - 4x$$

solve the equation

$$g(f(x)) = 0$$

A

$$\left[-\frac{a}{3} - \frac{\sqrt{4a^2 + 57}}{6},\ -\frac{a}{3} + \frac{\sqrt{4a^2 + 57}}{6}\right]$$

B

$$\left[-\frac{a}{3} - \frac{\sqrt{4a^2 + 9}}{6},\ -\frac{a}{3} + \frac{\sqrt{4a^2 + 9}}{6}\right]$$

C

$$\left[-\frac{a}{3} - \frac{\sqrt{4a^2 + 129}}{6},\ -\frac{a}{3} + \frac{\sqrt{4a^2 + 129}}{6}\right]$$

D

$$\left[-\frac{a}{3}-\frac{\sqrt{4a^2-111}}{6},\ -\frac{a}{3}+\frac{\sqrt{4a^2-111}}{6}\right]$$

(1 mark)

j If $f(x)$ is defined as

$$f(x)=3x-2$$

and $g(x)$ is defined as

$$g(x)=4x+2$$

solve the equation

$$g(f(x))+f(g(x))=2x$$

A

$$\frac{1}{11}$$

B

$$\frac{20}{29}$$

C

$$\frac{8}{17}$$

D

$$\frac{3}{5}$$

(1 mark)

k If $f(x)$ is defined as

$$f(x)=\sqrt{x+1}+5$$

what is the maximal domain of $f^{-1}(x)$?

A

$$[-1,\infty)$$

B

$$(5,\infty)$$

C

$$[5,\infty)$$

D

$$\mathbb{R}$$

(1 mark)

l If $f(x)$ is defined as

$$f(x)=e^{4-x}+3$$

what is the maximal domain of $f^{-1}(x)$?

A

$$(-\infty,3]$$

B

$$[3,\infty)$$

C

$$\mathbb{R}$$

D

$$(3,\infty)$$

(1 mark)

m If $f(x)$ is defined as

$$f(x) = \log(4x-8)+4$$

what is the maximal domain of $f^{-1}(x)$?

A

$$(-\infty,4)\cup(4,\infty)$$

B

$$(-\infty,\infty)$$

C

$$(2,\infty)$$

D

$$[4,\infty)$$

(1 mark)

n If $f(x)$ is defined as

$$f(x) = \frac{2x-1}{5-5x}$$

find $f^{-1}(x)$.

A

$$\left[\frac{x+2}{x+4}\right]$$

B

$$\left[\frac{5x+1}{5x+2}\right]$$

C

$$\left[\frac{5x-91}{5x-92}\right]$$

D

$$\left[\frac{25x+1}{25x+2}\right]$$

(1 mark)

o If $f(x)$ is defined as

$$f(x) = \sqrt{4-4x}$$

find $f^{-1}(x)$.

A

$$\left[1-36x^2\right]$$

B

$$\left[1-\frac{x^2}{1444}\right]$$

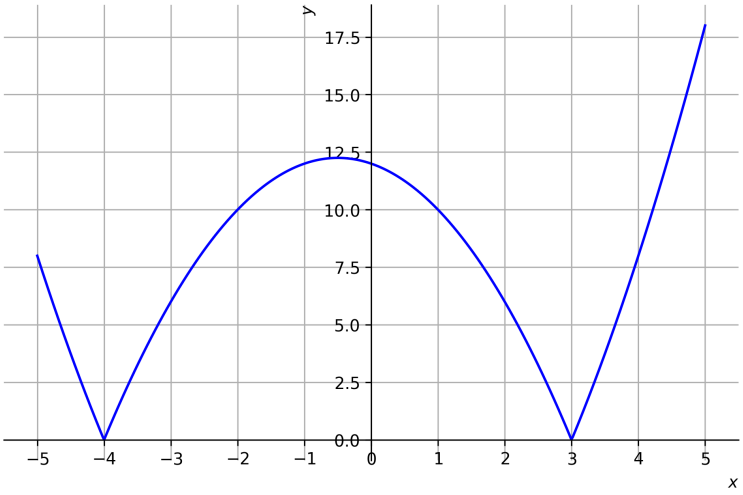
C

$$\left[1-\frac{x^2}{4}\right]$$

D

$$\left[1-\frac{(x-5)^2}{4}\right]$$

(1 mark)



Question 2

Let $f(x)=2^x-4$ and $g(x)=x+5$.

a State the domain of f. (1 mark)

b State the domain of g. (1 mark)

c State the range of f. (1 mark)

d Solve the equation $f(x)=g(x)$ and hence find the point(s) of intersection of the graphs of $y=f(x)$ and $y=g(x)$. (2 marks)

Question 3

Let $f(x) = k + x$ and $g(x) = 3^x + 1$, where k is a constant.

a State the domain of f. (1 mark)

b State the range of g. (1 mark)

c Find the inverse function $f^{-1}(x)$ in terms of k. (1 mark)

d Given that $f^{-1}(4) = 2$, find k. (2 marks)

Question 4

Consider the function $f : D \to R$ where the rule is $f(x) = 3 - 2\tan(3x - 2)$.

a State the maximal domain of f. (1 mark)

b State the period of f. (1 mark)

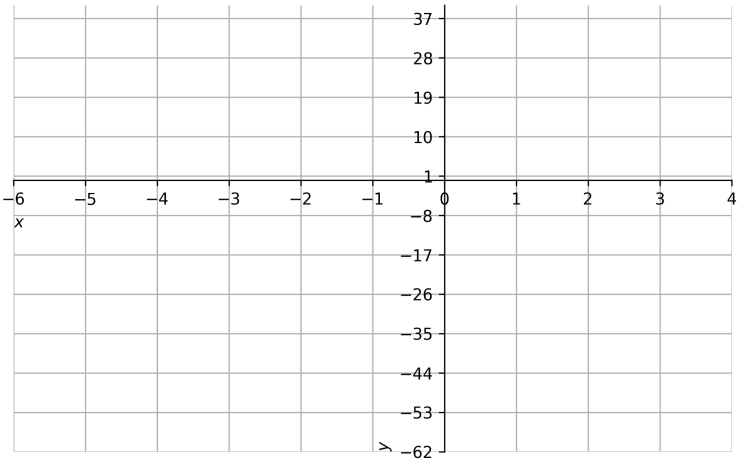
c State the range of f. (1 mark)

d Solve the equation $f(x) = \frac{2\sqrt{3}}{3} + 3$ for $x \in \mathbb{R}$. (3 marks)

Question 5

Let $f : R \to R$, $f(x) = x^3 + 2x^2 - 15x$.

Sketch the graph of $f(x)$ on the following grid.



(3 marks)

Question 6

Consider the function $f : D \to R$ where the rule is $f(x) = \frac{-5x-1}{5x-4}$. D is the maximal domain of the function.

a Express $f(x)$ in the form $A + \frac{B}{5x-4}$, where A and B are constants. (1 mark)

b Find the maximal domain of $f(x)$. (1 mark)

c Find the range of $f(x)$. (1 mark)

Question 7

Consider the function $f(x) = \sqrt{5-2x}$.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

c Find $f^{-1}(x)$. (1 mark)

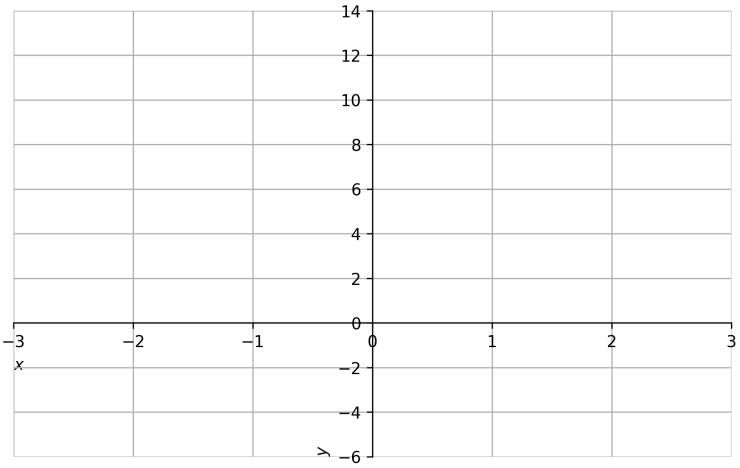
Question 8

A graph is defined by the rule $y = x^2 - 1$. The graph is transformed in the following order:

1. vertical dilation by a factor of 2
2. horizontal dilation by a factor of $\frac{1}{2}$
3. horizontal translation 1 units to the right

The transformed function is labeled Y.

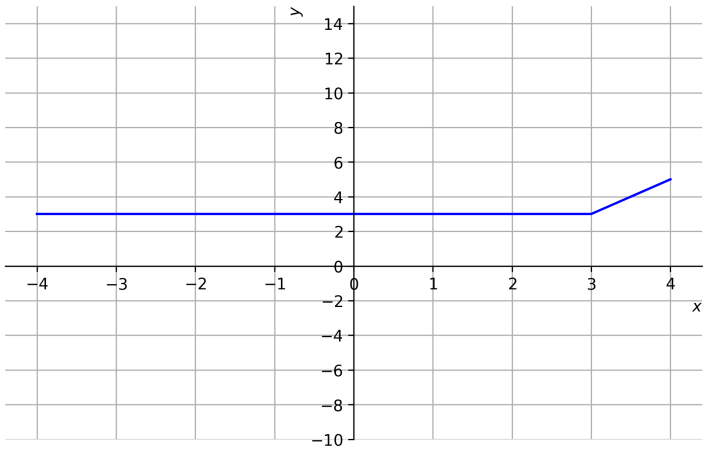
Sketch the graphs of y and Y in the following grid.



(4 marks)

Solutions

1. (a) D
   (b) D
   (c) True



   (d)
   (e) A
   (f) C
   (g) A
   (h) C
   (i) B
   (j) A

(k) C

(l) D

(m) B

(n) B

(o) C

2. (a) $\mathbb{R}$

(b) $\mathbb{R}$

(c) $(-4, \infty)$

(d) No simple exact real solution

3. (a) $\mathbb{R}$

(b) $(1, \infty)$

(c) $f^{-1}(x) = -k + x$

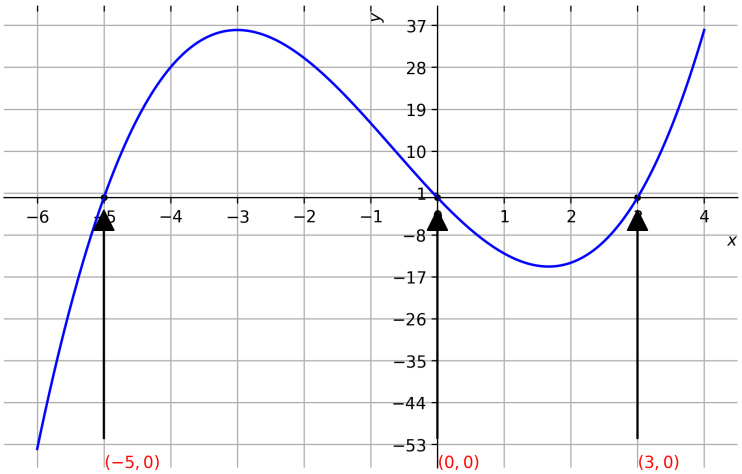
(d) 2

4. (a)
$$\mathbb{R} \setminus \left\{ \frac{\pi n}{3} + \frac{\pi}{6} + \frac{2}{3} : n \in \mathbb{Z} \right\}$$

(b) $\frac{\pi}{3}$

(c) $\mathbb{R}$

(d) $x = \frac{\pi n}{3} - \frac{\pi}{18} + \frac{2}{3},\ n \in \mathbb{Z}$

5. 

6. (a)
$$-1 - \frac{5}{5x - 4}$$

(b)
$$\left(-\infty, \frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$$

(c)
$$(-\infty, -1) \cup (-1, \infty)$$

7. (a)
$$\left(-\infty, \frac{5}{2}\right]$$

(b)

$$[0, \infty)$$

(c)

$$\frac{5}{2} - \frac{x^2}{2}$$

8.