MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
6	**6**	**38**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(10 marks)

a If $f(x)$ is defined as

$$f(x) = 4x - 1$$

and $g(x)$ is defined as

$$g(x) = 5x^2 - 2x + 3$$

use the chain rule to find $g'(f(x))$.

A

$$160x - 48$$

B

$$3200x - 960$$

C

$$5x^2 + 158x - 45$$

D

$$5x^2 - 6x + 4$$

(1 mark)

b If $f(x)$ is defined as

$$f(x) = x^2 - 5x - 1$$

and $g(x)$ is defined as

$$g(x) = 4x^2 + 3x + 4$$

use the chain rule to find $g'(f(x))$.

A

$$3x^2 + 8x + 5$$

B

$$16x^3 - 116x^2 + 193x + 29$$

C

$$16x^3 - 120x^2 + 190x + 25$$

D

$$96x^3 - 720x^2 + 1140x + 150$$

(1 mark)

c Differentiate the following expression with respect to u:

$$\left(u^4 + u^3 - 4u^2 + u + 1\right)^{\frac{10}{9}}$$

A

$$5u^2 - 5u - 3$$

B

$$\frac{10\left(-4u^3 - 3u^2 + 8u - 1\right)}{9\left(u^4 + u^3 - 4u^2 + u + 1\right)^{\frac{19}{9}}}$$

C

$$\frac{5\left(-4u^3 - 3u^2 + 8u - 1\right)}{54\left(u^4 + u^3 - 4u^2 + u + 1\right)^{\frac{19}{9}}}$$

D

$$\frac{190\left(-4u^3-3u^2+8u-1\right)}{9\left(u^4+u^3-4u^2+u+1\right)^{\frac{19}{9}}}$$

(1 mark)

d Does the following sketch represent the graph of the function

$$f(x)=|(x-3)(x+3)|$$

?

(1 mark)

e Differentiate the following expression with respect to v

$$\frac{-v-3}{\log\left(5v^2-v-3\right)}$$

A

$$\frac{-(v+3)(10v-1)+\left(5v^2-v-3\right)\log\left(5v^2-v-3\right)}{13\left(-5v^2+v+3\right)\log\left(5v^2-v-3\right)^2}$$

B

$$\frac{-(v+3)(10v-1)+\left(5v^2-v-3\right)\log\left(5v^2-v-3\right)}{\left(-5v^2+v+3\right)\log\left(5v^2-v-3\right)^2}$$

C

$$\frac{18\left(-(v+3)(10v-1)+\left(5v^2-v-3\right)\log\left(5v^2-v-3\right)\right)}{\left(-5v^2+v+3\right)\log\left(5v^2-v-3\right)^2}$$

D

$$4v^3-2v^2-5v-2$$

(1 mark)

f Find the expression for

$$\frac{dy}{dx}$$

for the following:

$$\frac{y(19x-20)}{20}=-\frac{19x^3}{10}+\frac{29x^2}{5}-\frac{23x}{20}-3$$

A

$$4-4x$$

B

$$\frac{2}{5}-\frac{2x}{5}$$

C

$$20-20x$$

D

$$-3x^3-x^2-3x-4$$

(1 mark)

g Differentiate the polynomial with respect to w

$$\frac{2w^4+4w^3-5w^2+5w+2}{w^{12}}$$

A

$$w^4 + 2w^3 - 5w^2 - 4w + 5$$

B

$$\frac{5\left(-16w^4 - 36w^3 + 50w^2 - 55w - 24\right)}{w^{13}}$$

C

$$\frac{-16w^4 - 36w^3 + 50w^2 - 55w - 24}{10w^{13}}$$

D

$$\frac{-16w^4 - 36w^3 + 50w^2 - 55w - 24}{w^{13}}$$

(1 mark)

h Differentiate the following expression with respect to m

$$\frac{5m-1}{\sqrt{5m^2+5m+3}}$$

A

$$\frac{35\,(m+1)}{2\,(5m^2+5m+3)^{\frac{3}{2}}}$$

B

$$\frac{105\,(m+1)}{(5m^2+5m+3)^{\frac{3}{2}}}$$

C

$$\frac{35\,(m+1)}{38\,(5m^2+5m+3)^{\frac{3}{2}}}$$

D

$$-2m^3 - 5m^2 - m - 1$$

(1 mark)

i Sketch the graph of the following function on the interval [-9, 9], labelling the intercepts.

$$f(x) = |\left|5\sin\left(\frac{2x}{3} - 6\right) + 3\right||$$

(1 mark)

j Differentiate the following expression with respect to t

$$2e^{3s^2t^2}\tan^2(2st)$$

A

$$228s^2te^{3s^2t^2}\tan^2(2st) + 152se^{3s^2t^2}\tan^3(2st) + 152se^{3s^2t^2}\tan(2st)$$

B

$$\frac{12s^2te^{3s^2t^2}\tan^2(2st)}{11} + \frac{8se^{3s^2t^2}\tan^3(2st)}{11} + \frac{8se^{3s^2t^2}\tan(2st)}{11}$$

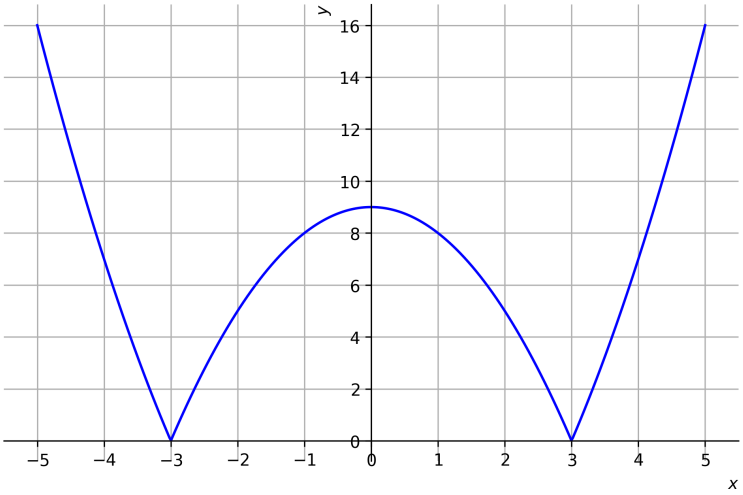
C

$$12s^2te^{3s^2t^2}\tan^2(2st) + 8se^{3s^2t^2}\tan^3(2st) + 8se^{3s^2t^2}\tan(2st)$$

D

$$-3e^{2s^2t^2}\cos(4st)$$

(1 mark)



Question 2

A festival program has 10 acts. A running order of 8 different acts is chosen. Two of the acts are the dance act and the choir act.

a Using permutations, how many different running orders are possible? (1 mark)

b How many running orders include both the dance act and the choir act, with the dance act before the choir act and exactly one act between them? (3 marks)

c How many running orders place both the dance act and the choir act in odd-numbered positions? (3 marks)

d How many running orders exclude both the dance act and the choir act? (2 marks)

Question 3

A fair spinner numbered from 1 to 5 is spun, and one token is drawn from a bag containing 6 blue tokens and 8 red tokens. Let event A be "the spinner lands on an even number" and event B be "the token drawn is blue".

a Find $P(A)$ and $P(B)$. (1 mark)

b Find $P(A \cap B)$. (1 mark)

c Determine whether events A and B are independent or dependent. Justify your answer. (2 marks)

d Find the probability that exactly one of the events A and B occurs. (2 marks)

Question 4

One scratch card is selected at random from 104 cards. The cards are bronze or silver, and each card pays 0, 5, or 10 dollars. The bronze-card counts are 12, 16, and 14 for these payouts, and the silver-card counts are 18, 24, and 20. Let X denote the payout on the chosen card.

a What is the probability that the scratch card drawn is bronze and pays 10 dollars?(1 mark)

b Find the probability distribution of X. (2 marks)

c How many scratch cards award a prize of more than 0 dollars? (1 mark)

d What is the probability that $X = 0$? (1 mark)

Question 5

A box contains red, yellow, blue, and white marbles. The probabilities of drawing a red marble, a yellow marble, and a white marble are equal, and the probability of drawing a blue marble is 3 times the probability of drawing a white marble. One marble is drawn at random.

a Let the probability of drawing a white marble be x. Express the probabilities of drawing the other colours in terms of x. (1 mark)

b Use the fact that the total probability is 1 to find the value of x and hence determine the probability of each colour. (2 marks)

c Using combinations, how many different pairs of colours can be chosen from red, yellow, blue, and white? (1 mark)

d What is the probability of drawing a marble that is not yellow? (1 mark)

Question 6

A continuous random variable X has a probability density function

$$p(x) = \frac{e^x}{7}, 0 \leq x \leq \log_e(8)$$

and zero otherwise.

(3 marks)

a Evaluate the mean.

(1 mark)

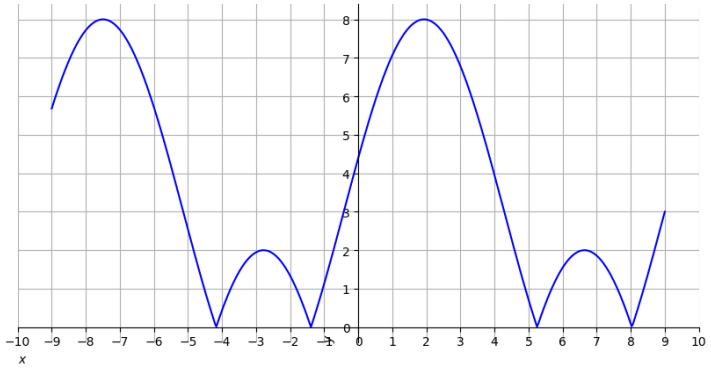
b Evaluate Pr($\log_e(6) \leq X \leq \log_e(11)$).

(1 mark)

c Evaluate Pr($X < \log_e(2)$).

(1 mark)

Solutions

1. (a) A
   (b) C
   (c) B
   (d) True
   (e) B
   (f) A
   (g) D
   (h) A
   (i) 
   (j) C

2. (a) Choose and arrange 8 acts from the 10 available acts. Therefore ${}^{10}P_8 = 1814400$.
   (b) Choose the starting position of the dance act so there is exactly one act between the two named acts. Then fill the remaining positions from the other acts. Therefore $6 \times {}^{8}P_6 = 120960$.
   (c) There are four odd-numbered positions in the running order. Choose and order two of them for the named acts, then fill the other positions from the remaining acts. Therefore ${}^{4}P_2 \times {}^{8}P_6 = 241920$.
   (d) Exclude the two named acts, then choose and arrange the full running order from the remaining acts. Therefore ${}^{8}P_8 = 40320$.

3. (a) $P(A) = 0.4,\ P(B) = \frac{3}{7}$
   (b) $\frac{6}{35}$
   (c) Independent, since $P(A \cap B) = 0.171429 = P(A)P(B)$
   (d) $\frac{17}{35}$

4. (a) $\frac{7}{52}$
   (b) $P(X = 0) = 0.288462,\ P(X = 5) = \frac{5}{13},\ P(X = 10) = 0.326923$
   (c) 74
   (d) $\frac{15}{52}$

5. (a) $P(\text{red}) = x,\ P(\text{yellow}) = x,\ P(\text{blue}) = 3x$
   (b) $x = \frac{1}{6},\ P(\text{red}) = \frac{1}{6},\ P(\text{yellow}) = \frac{1}{6},\ P(\text{blue}) = 0.5,\ P(\text{white}) = \frac{1}{6}$
   (c) 6
   (d) $\frac{5}{6}$

6. (a)
$$-1 + \frac{24\log(2)}{7}$$

(b)

$$\frac{2}{7}$$

(c)

$$\frac{1}{7}$$