

MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	35

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1**(9 marks)**a If $f(x)$ is defined as

$$f(x) = 5 - 4x$$

 $g(x)$ is defined as

$$g(x) = 2x^2 + 5x + 3$$

and $h(x)$ is defined as

$$h(x) = 2x^2 + 2x + 1$$

use the chain rule to find $g'(f(h(x)))$.

A

$$512x^3 + 768x^2 + 112x - 72$$

B

$$512x^3 + 770x^2 + 117x - 69$$

C

$$3072x^3 + 4608x^2 + 672x - 432$$

D

$$2x^2 + 9x - 2$$

(1 mark)

b Differentiate the following expression with respect to a :

$$\cos(\sin(a^2 + 2a + 1))$$

A

$$7(-2a - 2) \sin(\sin(a^2 + 2a + 1)) \cos(a^2 + 2a + 1)$$

B

$$3a^2 - a - 5$$

C

$$(-2a - 2) \sin(\sin(a^2 + 2a + 1)) \cos(a^2 + 2a + 1)$$

D

$$\frac{(-2a - 2) \sin(\sin(a^2 + 2a + 1)) \cos(a^2 + 2a + 1)}{14}$$

(1 mark)

c Sketch the graph of the function

$$f(x) = |x^3 - x^2 - 5x - 3|$$

(1 mark)

d Differentiate the following expression with respect to t

$$\frac{-3t - 5}{t^2 - 5t + 2}$$

A

$$\frac{3t^2 + 10t - 31}{t^4 - 10t^3 + 29t^2 - 20t + 4}$$

B

$$\frac{3t^2 + 10t - 31}{12(t^4 - 10t^3 + 29t^2 - 20t + 4)}$$

C

$$-4t^3 + t^2 + 4t - 1$$

D

$$\frac{18(3t^2 + 10t - 31)}{t^4 - 10t^3 + 29t^2 - 20t + 4}$$

(1 mark)

e Differentiate the polynomial with respect to a

$$\frac{2a^3 - 5a^2 + a + 5}{a^4}$$

A

$$\frac{-2a^3 + 10a^2 - 3a - 20}{10a^5}$$

B

$$\frac{5(-2a^3 + 10a^2 - 3a - 20)}{a^5}$$

C

$$-4a^3 - 2a^2 - 5a + 2$$

D

$$\frac{-2a^3 + 10a^2 - 3a - 20}{a^5}$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = |x - 4|$$

evaluate $f(-2)$.

A 8

B 7

C 6

D 5

(1 mark)

g Differentiate the following expression with respect to l

$$\sin^2(3l^2m) + 5\cos(2lm)$$

A

$$4\sin(3m)$$

B

$$20m(3l\sin(6l^2m) - 5\sin(2lm))$$

C

$$2m(3l\sin(6l^2m) - 5\sin(2lm))$$

D

$$\frac{2m(3l\sin(6l^2m) - 5\sin(2lm))}{13}$$

(1 mark)

h If $f(x)$ is defined as

$$f(x) = k + 8x^3 - 36x^2 + 54x - 30$$

for what value of k will $f(x)$ be invertible? Hint: A cubic function is invertible only if it has a single stationary point.

- A -6
- B -8
- C -4
- D 3

(1 mark)

i Let

$$f'(x) = -16x^3 + 6x^2 - 4x + 2$$

If $f(9) = -24934$, find $f(x)$.

A

$$f(x) = -3x^3 + 4x^2 + x - 1$$

B

$$f(x) = -\frac{4x^4}{11} + \frac{2x^3}{11} - \frac{2x^2}{11} + \frac{2x}{11} - \frac{4}{11}$$

C

$$f(x) = -20x^4 + 10x^3 - 10x^2 + 10x - 20$$

D

$$f(x) = -4x^4 + 2x^3 - 2x^2 + 2x - 4$$

(1 mark)

Question 2

A florist is making a bouquet of 5 flowers from 18 distinct flowers: 6 roses, 5 lilies, 4 daisies, and 3 tulips. Two of the roses are marked as featured stems.

a Using combinations, how many different bouquets of 5 flowers can be made? (1 mark)

b How many bouquets contain exactly 2 roses, 1 lily, 1 daisy, and 1 tulip? (2 marks)

c How many bouquets contain at least one tulip? (2 marks)

- d How many bouquets contain no roses? (1 mark)

Question 3

A security code has 4 different characters chosen from the letters E, H, M, U, Y, Z and the digits 3, 4, 6, 7. No character is repeated.

- a Using permutations, how many such codes are possible? (1 mark)

- b How many codes start with a letter and end with a digit? (2 marks)

- c How many codes contain exactly 2 letters and 2 digits? (3 marks)

- d How many codes end with a digit and contain the letter E ? (3 marks)

Question 4

A fair coin is tossed and a fair six-sided die is rolled. Let event A be "the coin shows heads" and event B be "the die shows an even prime number".

- a Find $P(A)$ and $P(B)$. (1 mark)

b Find $P(A \cap B)$. (1 mark)

c Determine whether events A and B are independent or dependent. Justify your answer. (2 marks)

d Find $P(A \mid B)$. (1 mark)

Question 5

From a box containing 8 batteries, including 2 faulty ones, two batteries are chosen without replacement.

a What is the probability that both batteries selected are faulty? (1 mark)

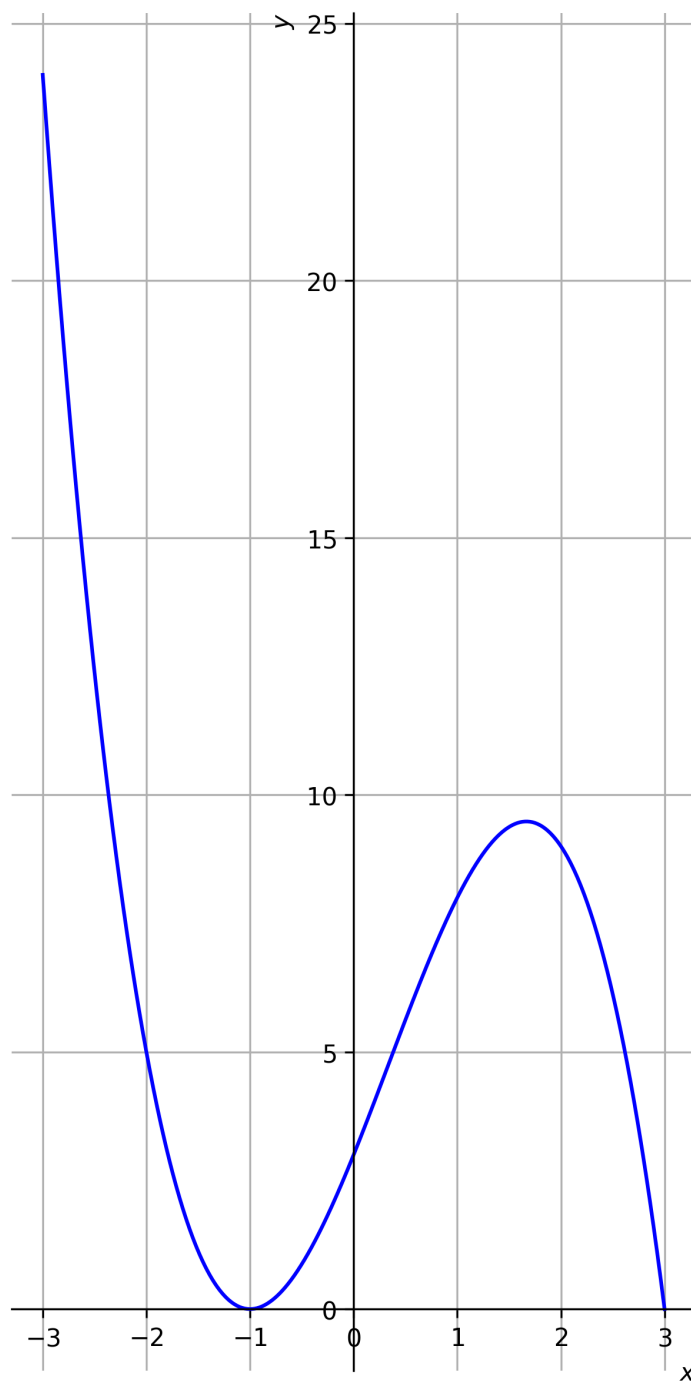
b What is the probability that the first battery is working and the second is faulty? (2 marks)

c What is the probability that at least one of the selected batteries is faulty? (2 marks)

d What is the probability that both selected batteries are working? (1 mark)

Solutions

1. (a) A
(b) C



- (c)
- (d) A
- (e) D
- (f) C
- (g) C
- (h) D
- (i) D

2. (a) $\binom{18}{5} = 8568$
 (b) $\binom{6}{2} \times \binom{5}{1} \times \binom{4}{1} \times \binom{3}{1} = 900$
 (c) $\binom{18}{5} - \binom{15}{5} = 5565$

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- (d) $\binom{12}{5} = 792$
3. (a) ${}^{10}P_4 = 5040$
(b) $6 \times 4 \times {}^8P_2 = 1344$
(c) $\binom{4}{2} \times {}^6P_2 \times {}^4P_2 = 2160$
(d) $3 \times 4 \times {}^8P_2 = 672$
4. (a) $P(A) = 0.5, P(B) = \frac{1}{3}$
(b) $\frac{1}{6}$
(c) Independent, since $P(A \cap B) = \frac{1}{6} = P(A)P(B)$
(d) $\frac{1}{2}$
5. (a) $\frac{1}{28}$
(b) $\frac{3}{14}$
(c) $\frac{13}{28}$
(d) $\frac{15}{28}$