MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
7	**7**	**68**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(7 marks)

a Solve the following inequality

$$t^2+4t-21 \geq 0$$

A

$$-3 < t < -1$$

B

$$t \leq -7, t \geq 3$$

C

$$t < -7, t > -1$$

D

$$t < -8, t > 8$$

(1 mark)

b Find the value of a that will make the polynomial

$$ax-10x^3+18x^2-20$$

divisible by

$$4-5x$$

A -9
B 3
C -12
D 17

(1 mark)

c Does the following sketch represent the graph of the function

$$f(x)=|x+1|$$

? (1 mark)

d If $f(x)$ is defined as

$$f(x)=4x-2$$

$g(x)$ is defined as

$$g(x)=-x^2+2x+5$$

and $h(x)$ is defined as

$$h(x)=-x^2+4x-5$$

use the chain rule to find $g^{'}(f(h(x)))$.

A

$$-64x^3+383x^2-878x+741$$

B

$$-896x^3+5376x^2-12320x+10304$$

C

$$-x^2-2x+7$$

D

$$-64x^3+384x^2-880x+736$$

(1 mark)

e Differentiate the following expression with respect to q

$$-4\log\left(-q^2-q-4\right)$$

A

$$-2q^3-5q^2+2q+5$$

B

$$\frac{20\left(-2q-1\right)}{q^2+q+4}$$

C

$$\frac{4\left(-2q-1\right)}{q^2+q+4}$$

D

$$\frac{-2q-1}{3\left(q^2+q+4\right)}$$

(1 mark)

f For the function

$$f(x)=\frac{x^3}{3}+3x^2+14x-4$$

find the two points on the graph of $f(x)$ at which the tangents of the graph are parallel to the line connecting the two points $(-4,-5)$ and $(0,15)$.

A -3 and -3

B 16 and -2

C There are no solutions

D -11 and -6

(1 mark)

g For the curve defined by

$$2x^3+x^2+4x+2$$

find the equation of the tangent line at $x=-5$.

A

$$y=24x-123$$

B

$$y=2448x+11997$$

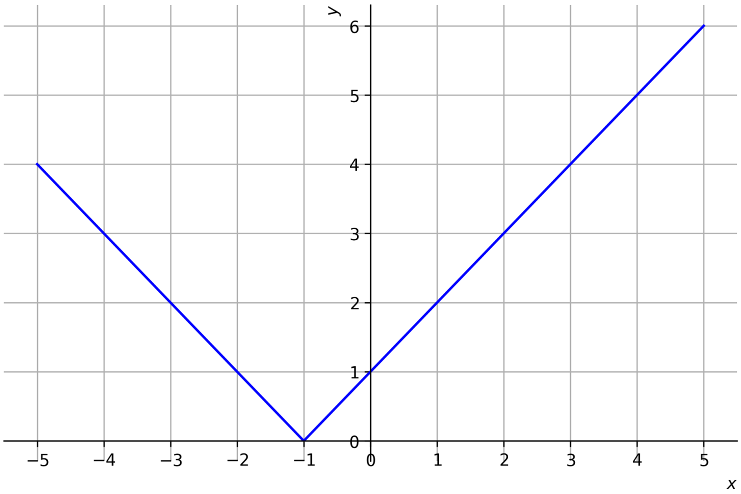
C

$$y=144x+477$$

D

$$y=144x-54$$

(1 mark)



Question 2

a A vibration sensor is attached to a reference marker on a laboratory calibration platform. For the first trial, the marker's vertical displacement from its centre line is modelled by

$$y(t) = \frac{3\cos\left(5\sqrt{10}t\right)}{20}.$$

Here t is measured in seconds and y in metres. This trial is called Model A.

i Differentiate $y(t)$ to find $y'(t)$. (1 mark)

ii Find the first value of $t > 0$ for which $y(t) = 0$. (2 marks)

iii Determine the maximum speed during the first full period, and state the times at which it occurs. (2 marks)

iv Differentiate again to find $y''(t)$, then evaluate $y''(0.10)$. (2 marks) v State the amplitude and period of Model A. (1 mark)

b The same calibration platform is then run at a faster setting while keeping the same maximum displacement. The displacement is modelled by

$$y(t) = \frac{3\cos(20t)}{20}.$$

This trial is called Model B.

i State the amplitude and period of Model B. (1 mark)

ii Express the period of Model B as a percentage of the period of Model A, correct to one decimal place. (2 marks)

iii Differentiate $y(t)$ to find $y'(t)$, then determine the maximum speed during the first full period and the times at which it occurs. (2 marks)

iv Find the greatest magnitude of $y''(t)$ for Model B and compare it with the corresponding value for Model A. (2 marks)

v State the range of Model B. (1 mark)

c A lighter sensor mount is tested next. The recorded displacement has a smaller maximum displacement and a different period:

$$y(t) = \frac{\cos\left(8\sqrt{5}t\right)}{10}.$$

This trial is called Model C.

i Express the period of Model C as a percentage of the period of Model B, correct to one decimal place. (2 marks)

ii Find the first value of $t > 0$ for which $y(t) = 0$. (2 marks)

iii Find the maximum speed and the greatest magnitude of $y''(t)$. (2 marks)

iv State the amplitude and range of Model C. (1 mark)

d In a final trial, the marker starts at the centre line and initially moves upward. Its displacement is modelled by

$$y(t) = \frac{\sin(20t)}{10}.$$

This trial is called Model D.

i Differentiate to obtain expressions for $y'(t)$ and $y''(t)$. (2 marks)

ii Find the first time after $t = 0$ at which $y(t)$ reaches its maximum value, and state that value. (2 marks) iii Find the first positive value of t for which $y(t) = 0$ again, and give the corresponding value of $y'(t)$. (2 marks)

iv State the amplitude and period of Model D. (1 mark)

v Determine the maximum magnitude of $y''(t)$. (1 mark)

Question 3

Let $f(x) = 2x^3 - 6x^2 - 6$ and $g(x) = x^3 - 9x^2 + 4x - 6$.

a State the domain of f. (1 mark)

b Differentiate $f(x)$ to find $f'(x)$. (1 mark)

c Find the equation of the tangent line to $y = f(x)$ at $x = -2$. (2 marks) d Use the second derivative to find the point of inflection of f. (2 marks)

Question 4

For $-1 \leq a \leq 3$, let R be the region between the curves

$$y = 11 - (x-1)^2 \quad \text{and} \quad y = 2$$

from $x = a-1$ to $x = a+1$.

a Find an expression for the area $A(a)$ of the region. (2 marks)

b Differentiate $A(a)$ with respect to a. (1 mark)

c Find the value of a for which the area is maximised. (2 marks)

d Hence find the maximum area. (1 mark)

Question 5

A distillery collects botanical extract at a rate, in millilitres per minute, modelled by

$$\frac{dV}{dt} = 60e^{-\frac{t}{4}},$$

where t is the time in minutes after collection begins. Initially, there are 30 millilitres of extract in the flask.

a Find the total volume of extract collected during the first 4 minutes. (2 marks)

b Find an expression for the volume $V(t)$ of extract in the flask after t minutes. (2 marks)

c Determine the volume of extract in the flask after 4 minutes. (1 mark)

d Find the average collection rate over the first 4 minutes. (1 mark)

Question 6

Consider the function $f(x) = \frac{-4x-5}{3x+4}$.

a Express $f(x)$ in the form $A + \frac{B}{3x+4}$, where A and B are constants. (1 mark)

b Find the maximal domain of $f(x)$. (1 mark) c Find the range of $f(x)$. (1 mark)

d If another function $g(x)$ is defined as

$$g(x) = -5x - 3$$

solve the equation

$$g(f(x)) = 0$$

(2 marks)

Question 7

Points P and Q lie on the graph of $y = f(x)$, where $f(x) = x^2 - 4x + 5$, at $x = -5$ and $x = -1$ respectively.

a State the coordinates of P and Q. (1 mark)

b Find the equation of the line segment PQ. (2 marks)

c State the coordinates of the turning point of the graph. (1 mark) d Find the exact area enclosed by the graph of $y = f(x)$ and the line segment PQ.(3 marks)

Solutions

1. (a) B
   (b) D
   (c) True
   (d) D
   (e) C
   (f) A
   (g) C

2. (a) (i) Differentiate the given function to obtain $y'(t) = -\frac{3\sqrt{10}\sin\left(5\sqrt{10}t\right)}{4}$ m s^{-1}.
   (ii) Using the required working, the answer is $t = \frac{\sqrt{10}\pi}{100} \approx 0.0993$ s.
   (iii) Using the required working, the answer is maximum speed $= \frac{3\sqrt{10}}{4} \approx 2.37$ m s^{-1}, at $t = \frac{\sqrt{10}\pi}{100}$ and $t = \frac{3\sqrt{10}\pi}{100}$.
   (iv) Differentiate the given function to obtain $y''(t) = -\frac{75\cos\left(5\sqrt{10}t\right)}{2}, \quad y''(0.10) = 0.39.$
   (v) Using the required working, the answer is Amplitude

$$= 0.15 \text{ m}, \qquad T = \frac{\sqrt{10}\pi}{25} \approx 0.3974 \text{ s}$$

.

(b) (i) Using the required working, the answer is Amplitude

$$= 0.15 \text{ m}, \qquad T = \frac{\pi}{10} \approx 0.314 \text{ s}$$

.

   (ii) Using the required working, the answer is $T_B = \frac{\pi}{10} \approx 0.314$ s, which is 79.1% of the period of Model A..
   (iii) Differentiate the given function to obtain $y'(t) = -3\sin(20t)$, maximum speed $= 3.00$ m s^{-1} at $t = \frac{\pi}{40}$ and $t = \frac{3\pi}{40}$.
   (iv) Using the required working, the answer is greatest magnitude of $y''(t)$ for Model B $= 60 = 60$ m s^{-2}, while for Model A it is $\frac{75}{2} = 37.5$ m s^{-2}. So Model B has the larger maximum second derivative..
   (v) Determine the possible output values from the function. The range is $-0.15 \le y(t) \le 0.15$.

(c) (i) Using the required working, the answer is $T = \frac{\sqrt{5}\pi}{20} \approx 0.3512$ s, which is 111.8% of the period of Model B..
   (ii) Using the required working, the answer is $t = \frac{\sqrt{5}\pi}{80} \approx 0.0878$ s.
   (iii) Using the required working, the answer is maximum speed $= \frac{4\sqrt{5}}{5} \approx 1.79$ m s^{-1}, greatest magnitude of $y''(t) = 32 \approx 32$ m s^{-2}.

(iv) Using the required working, the answer is Amplitude

$$= 0.10 \text{ m}, \qquad -0.10 \leq y(t) \leq 0.10$$

.

(d) (i) Differentiate the given function to obtain

$$y'(t) = 2\cos(20t), \qquad y''(t) = -40\sin(20t)$$

.

(ii) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives

$$t = \frac{\pi}{40} \approx 0.0785 \text{ s}, \qquad y_{\max} = \frac{1}{10} \text{ m}$$

.

(iii) Using the required working, the answer is

$$t = \frac{\pi}{20} \approx 0.1571 \text{ s}, \qquad y'(t) = -2 \text{ m s}^{-1}$$

.

(iv) Using the required working, the answer is Amplitude

$$= 0.1 \text{ m}, \qquad T = \frac{\pi}{10} \approx 0.314 \text{ s}$$

.

(v) Using the required working, the answer is maximum magnitude of $y''(t)$= $40.0 = 40$ m s^{-2}.

3. (a) Identify all allowable input values. The domain is $\mathbb{R}$.
   (b) Differentiate the given function to obtain $6x^2 - 12x$.
   (c) Find the gradient from the derivative at the given point, then use point-slope form. This gives $y = 48x + 50$.
   (d) Differentiate the first derivative once more to obtain $(1, -10)$.

4. (a) $A(a) = -2a^2 + 4a + \frac{46}{3}$
   (b) $A'(a) = 4 - 4a$
   (c) $a = 1$
   (d) $A_{max} = \frac{52}{3}$

5. (a) Using the required working, the answer is $240 - \frac{240}{e}$.
   (b) Using the required working, the answer is $V(t) = 270 - 240e^{-\frac{t}{4}}$.
   (c) Using the required working, the answer is 181.71.
   (d) Using the required working, the answer is 37.93.

6. (a)

$$-\frac{4}{3} + \frac{1}{3(3x+4)}$$

(b)

$$\left(-\infty, -\frac{4}{3}\right) \cup \left(-\frac{4}{3}, \infty\right)$$

(c)

$$\left(-\infty, -\frac{4}{3}\right) \cup \left(-\frac{4}{3}, \infty\right)$$

(d)

$$\left[-\frac{13}{11}\right]$$

7. (a) $(-5, 50)$ and $(-1, 10)$
   (b) $y = -10x$
   (c) $(2, 1)$
   (d) $\frac{32}{3}$