MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	**5**	**29**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(5 marks)

a If $f(x)$ is defined as

$$f(x) = \frac{1}{3-x}$$

$g(x)$ is defined as

$$g(x) = -4x^2 - 5x + 5$$

and $h(x)$ is defined as

$$h(x) = 3x - 5$$

use the chain rule to find $g^{'}(f(h(x)))$.

A

$$-4x^2 - 5x + 5 - \frac{24}{-27x^3 + 216x^2 - 576x + 512} - \frac{15}{9x^2 - 48x + 64}$$

B

$$-\frac{48}{-27x^3 + 216x^2 - 576x + 512} - \frac{30}{9x^2 - 48x + 64}$$

C

$$-4x^2 - 5x + 5 - \frac{1}{3-x}$$

D

$$\frac{9(16-5x)}{(3x-8)^3}$$

(1 mark)

b Differentiate the following expression with respect to u

$$-4e^{8s^2u^2}\tan\left(2su^2\right)$$

A

$$-176su\left(4s\tan\left(2su^2\right) + \frac{1}{\cos^2\left(2su^2\right)}\right)e^{8s^2u^2}$$

B

$$-16su\left(4s\tan\left(2su^2\right) + \frac{1}{\cos^2\left(2su^2\right)}\right)e^{8s^2u^2}$$

C

$$-\frac{8su\left(4s\tan\left(2su^2\right) + \frac{1}{\cos^2\left(2su^2\right)}\right)e^{8s^2u^2}}{3}$$

D

$$6e^{2s^2u^2}\tan\left(4su^2\right)$$

(1 mark)

c Sketch the graph of the function

$$f(x) = x + |x-1| - 1$$

(1 mark)

d Differentiate the following expression with respect to z

$$15u^2z\tan\left(3u^3z^3\right) + 3\sin\left(2z^3\right)$$

A

$$5\cos^2(5u)$$

B

$$\frac{135u^5z^3}{\cos^2(3u^3z^3)} + 15u^2\tan\left(3u^3z^3\right) + 18z^2\cos\left(2z^3\right)$$

C

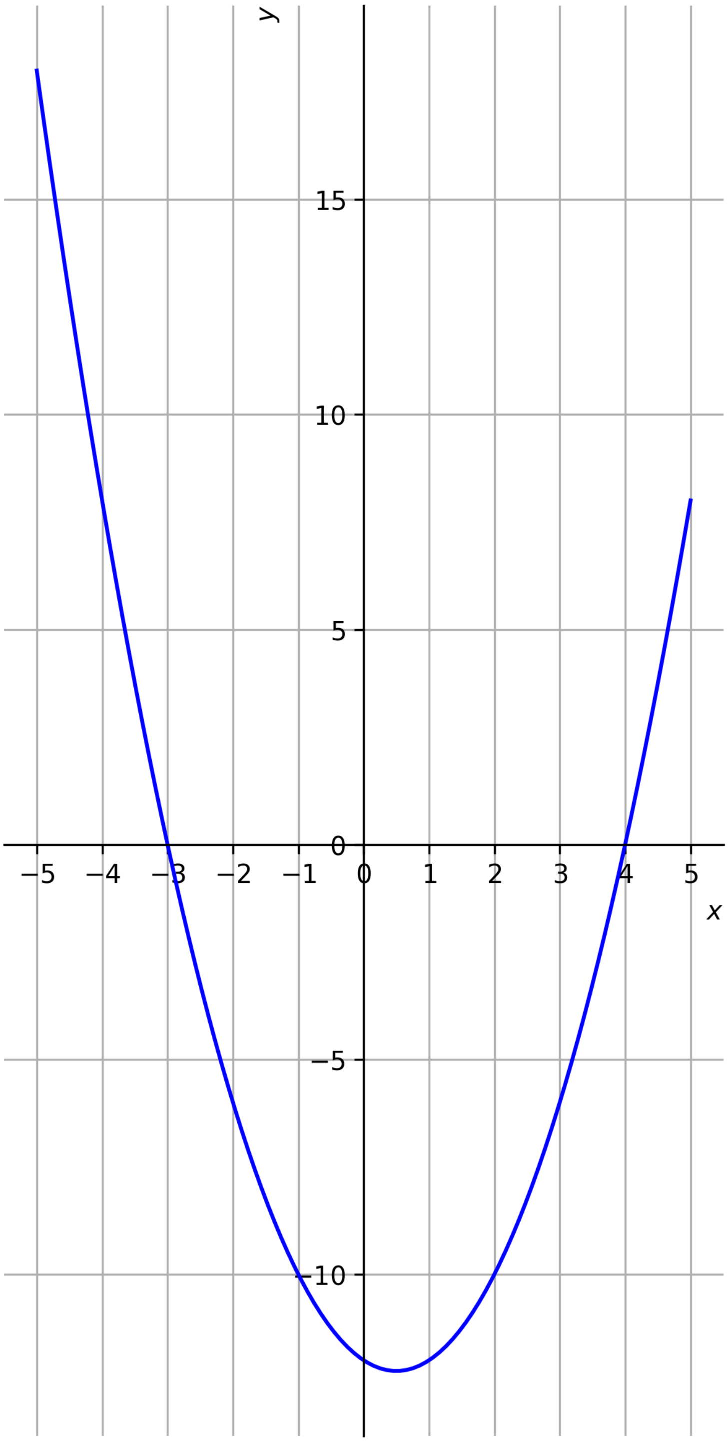
$$\frac{2430u^5z^3}{\cos^2(3u^3z^3)} + 270u^2\tan\left(3u^3z^3\right) + 324z^2\cos\left(2z^3\right)$$

D

$$\frac{135u^5z^3}{19\cos^2(3u^3z^3)} + \frac{15u^2\tan\left(3u^3z^3\right)}{19} + \frac{18z^2\cos\left(2z^3\right)}{19}$$

(1 mark)

e What is the sign of the tanget at point -5 in the following graph? (1 mark)



Question 2

A ball is thrown upwards and its height (in metres) above the ground after t seconds is given by:

$$h(t) = -t^2 + 8t - 16,$$

where $t \geq 0$.

a Find the expression for the ball's vertical velocity at any time t. (1 mark) b Determine the time at which the ball reaches its maximum height. (1 mark)

c Calculate the maximum height reached by the ball. (1 mark)

d When does the ball hit the ground? (1 mark)

Question 3

An observation wheel measures a passenger's height above the ground in metres by the function

$$h(t) = 22 + 14\sin\left(\frac{\pi t}{12}\right),$$

where t is the time in minutes after the ride starts.

a Differentiate $h(t)$ to find the passenger's vertical velocity function $\frac{dh}{dt}$. (1 mark)

b Calculate the passenger's vertical velocity after 4 minutes and interpret the sign of your answer. (2 marks) c Determine the first time the passenger reaches the highest point of the ride. (2 marks)

d During the first revolution, when is the passenger descending most rapidly, and what is the corresponding vertical velocity? (2 marks)

Question 4

A spring-mounted platform moves horizontally from its equilibrium position according to

$$s(t) = \frac{2}{5}\sin(3t),$$

where $s(t)$ is the displacement in metres and t is the time in seconds.

a Differentiate $s(t)$ to find the velocity function $v(t)$. (1 mark)

b Find the velocity of the platform at $t = \frac{\pi}{9}$ seconds. (2 marks)

c Determine the first positive time the platform passes through its equilibrium position. (2 marks)

d Find the maximum speed of the platform and the maximum magnitude of its acceleration. (2 marks)

Question 5

Suppose

$$f'(x) = x^3 - 5x^2 + 4x$$

a Solve $f'(x) = 0$ to find the possible x-coordinates of stationary points. (1 mark)

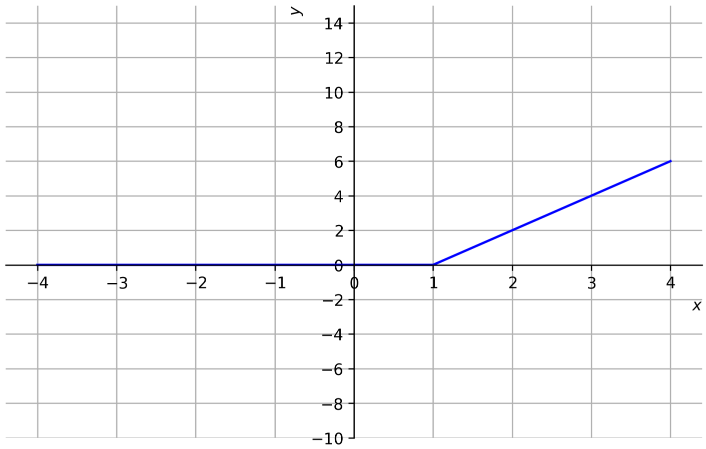
b Differentiate $f'(x)$ to find $f''(x)$. (1 mark)

c State the intervals on which f is increasing and the intervals on which f is decreasing. (2 marks)

d If $(7, -758.417)$ is a stationary point, find the expression of $f(x)$. (2 marks)

Solutions

1. (a) D
   (b) B

(c) 

(d) B

(e) negative

2. (a) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives 8 - 2 t.
   (b) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives 4.
   (c) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives 0.
   (d) Set the model equal to the required value and solve for the variable. This gives 4.

3. (a) Differentiate the given function to obtain $\frac{dh}{dt} = \frac{7\pi\cos\left(\frac{\pi t}{12}\right)}{6}$.
   (b) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives $\left.\frac{dh}{dt}\right|_{t=4} = \frac{7\pi}{12} \approx 1.83 \quad \text{m min}^{-1}$, so the passenger is moving upward..
   (c) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives $t = 6 \quad \text{minutes}$.
   (d) The passenger descends most rapidly at $t = 12$ minutes, with $\frac{dh}{dt} = -\frac{7\pi}{6} \approx -3.67 \quad \text{m min}^{-1}$.

4. (a) Differentiate the given function to obtain $v(t) = \frac{6\cos(3t)}{5}$.
   (b) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives $v\left(\frac{\pi}{9}\right) = \frac{3}{5} \approx 0.6 \quad \text{m s}^{-1}$.
   (c) Using the required working, the answer is $t = \frac{\pi}{3} \approx 1.047 \quad \text{s}$.
   (d) Using the required working, the answer is $v_{\max} = 1.2 \quad \text{m s}^{-1}, \quad$ maximum acceleration magnitude $= 3.6 \quad \text{m s}^{-2}$.

5. (a) Set the first derivative equal to 0 to find the stationary point, then classify it using the second derivative or sign changes. This gives $[0,\ 1,\ 4]$.
   (b) Differentiate the given function to obtain $3x^2 - 10x + 4$.
   (c) Using the required working, the answer is Increasing on $(0,1) \cup (4,\infty)$, decreasing on $(-\infty,0) \cup (1,4)$.
   (d) Integrate the given derivative to obtain f(x), then use the stationary point to determine the constant of integration. This gives

$$f(x) = \frac{x^4}{4} - \frac{5x^3}{3} + 2x^2 - 126x - 3$$

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