# MATHMATICAL METHODS

## Written examination: SAC

Reading time: 15 minutes
Writing time: 1 hour

## QUESTION AND ANSWER BOOK
## Structure of book

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| **9** | **9** | **49** |

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

**Instructions**

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

**At the end of the examination**

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

## Question 1

**(11 marks)**

a If

$$f(x) = -3 + \frac{1}{x-5}$$

and

$$g(x) = 5x + 1$$

let

$$h(x) = f(g(x))$$

. What is the domain of $h(x)$?

A

$$(-\infty, -3) \cup (-3, \infty)$$

B

$$\left(-\infty, -\frac{4}{5}\right) \cup \left(-\frac{4}{5}, \infty\right)$$

C

$$\left(-\infty, \frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$$

D

$$\mathbb{R}$$

(1 mark)

b If

$$f(x) = -6 + \frac{1}{x-6}$$

and

$$g(x) = 1 - x$$

let

$$h(x) = f(g(x))$$

What are the domain and range of $h(x)$?

A

Domain: $(-\infty, -5) \cup (-5, \infty)$, Range: $(-\infty, -6) \cup (-6, \infty)$

B

Domain: $(-\infty, 5) \cup (5, \infty)$, Range: $(-\infty, -6) \cup (-6, \infty)$

C

Domain: $(-\infty, -6) \cup (-6, \infty)$, Range: $(-\infty, -5) \cup (-5, \infty)$

D

Domain: $\mathbb{R}$, Range: $(-\infty, -6) \cup (-6, \infty)$

(1 mark)

c If $f(x)$ is defined as

$$f(x) = 2 - 3x$$

and $g(x)$ is defined as

$$g(x) = 5x^2 - x - 2$$

simplify $g(f(x))$.

A

$$20x^2 - 4x - 8$$

B

$$45x^2 - 57x + 16$$

C

$$5x^2 + 2x - 4$$

D

$$5x^2 - 4x$$

(1 mark)

d If $f(x)$ is defined as

$$f(x) = -3x - 5$$

and $g(x)$ is defined as

$$g(x) = -5x - 4$$

solve the equation

$$g(f(x)) + f(g(x)) = 0$$

A

$$\frac{17}{50}$$

B

$$\frac{4}{29}$$

C

$$\frac{23}{59}$$

D

$$-\frac{14}{15}$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = -4 + \frac{1}{1-5x}$$

what is the maximal domain of $f^{-1}(x)$?

A

$$\left(-\infty, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$$

B

$$(-4, \infty)$$

C

$$(-\infty, -4) \cup (-4, \infty)$$

D

$$\mathbb{R}$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = \sqrt{1-4x}$$

find $f^{-1}(x)$.

A

$$\left[\frac{1}{4} - \frac{x^2}{4}\right]$$

B

$$\left[\frac{1}{4}-\frac{(x-18)^2}{4}\right]$$

C

$$\left[\frac{1}{4}-\frac{x^2}{900}\right]$$

D

$$\left[\frac{1}{4}-25x^2\right]$$

(1 mark)

g If $f(x)$ is defined as

$$f(x)=\sqrt[3]{3x+4}$$

find $f^{-1}(x)$.

A

$$\left[\frac{512x^3}{3}-\frac{4}{3}\right]$$

B

$$\left[\frac{x^3}{3}-\frac{4}{3}\right]$$

C

$$\left[\frac{x^3}{10125}-\frac{4}{3}\right]$$

D

$$\left[-\frac{(x-5)^3}{3}-\frac{4}{3}\right]$$

(1 mark)

h If $f(x)$ is defined as

$$f(x)=\sqrt{5x-1}+4$$

what is the maximal range of $f^{-1}(x)$?

A

$$(-\infty,4]$$

B

$$[4,\infty)$$

C

$$\mathbb{R}$$

D

$$\left[\frac{1}{5},\infty\right)$$

(1 mark)

i A line is parallel to the graph

$$9x+5y-4=0$$

and passes through the point $(0,4)$. What is the equation of that line?

A

$$y=9x+5y-4$$

B
$$y = 4 - \frac{9x}{5}$$

C
$$y = 36 - \frac{81x}{5}$$

D
$$y = \frac{9x}{5} + 2$$

(1 mark)

j A line is perpendicular to the graph
$$-10x + 6y - 7 = 0$$
and passes through the point $(-7, 8.2)$. What is the equation of that line?

A
$$y = -10x + 6y - 7$$

B
$$y = \frac{3x}{5} - 3$$

C
$$y = 4 - \frac{3x}{5}$$

D
$$y = 32 - \frac{24x}{5}$$

(1 mark)

k For the following two functions,
$$f(x) = -2ax^2 - 3ax + 2$$
and
$$g(x) = a(3x - 1)$$
what values of $a$ will ensure that the graphs of these functions never intersect?

A
$$a < 19, a > 9$$

B
$$a > -\frac{4}{11}, a < 0$$

C
$$a > -16, a \leq 11$$

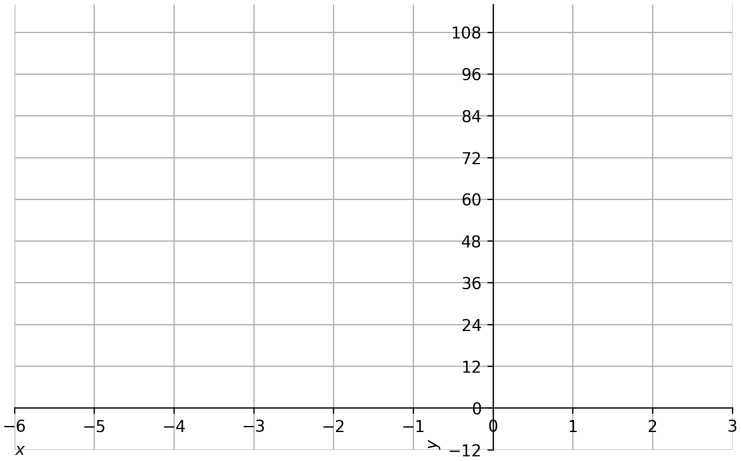
D
$$a > -9, a > 2$$

(1 mark)

## Question 2

Let $f : R \to R$, $f(x) = x^4 + 6x^3 - 5x^2 - 42x + 40$.

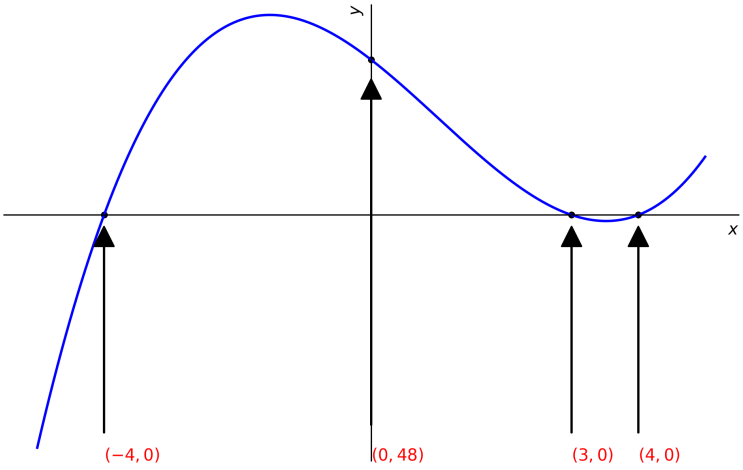
Sketch the graph of $f(x)$ on the following grid.




(4 marks)

## Question 3

Consider the following graph.




a Write the rule for the graph. (1 mark)

b This function is transformed according to the rule

$$g(x) = 3f(x-2)$$

Simplify $g(x)$ in terms of $x$. (2 marks)

c Find the maximal domain of $f(x)$. (1 mark)

d Another function $g(x)$ is defined as

$$g(x) = f(x)x^2 - 4x$$

Simplify $g(x)$ in terms of $x$. (1 mark)

## Question 4

Consider the function $f(x) = 20x^2 - 28x + 8$.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

c If the domain of the function is $[a, \infty)$, what values of $a$ will make the function one-to-one?(2 marks)

d What is the range of $f(x)$ if $x$ belongs to this interval? (2 marks)

## Question 5

Consider the function $f(x) = ax^2 - 3a + x$.

a What is the domain of $f(x)$ if $a = 6$? (1 mark)

b What is the range of $f(x)$ if $a = 1$? (1 mark)

c If another function $g(x)$ is defined as

$$g(x) = -4x - 2$$

solve the equation

$$g(f(x)) = 0$$

(2 marks)

d This parabola interset with the line $y = -3ax + 2$. Find the value(s) of $x$ at which the two graphs intersect. (3 marks)

## Question 6

Consider the function $f : D \to R$ where the rule is $f(x) = \sqrt{5x-4}$. $D$ is the maximal domain of the function.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

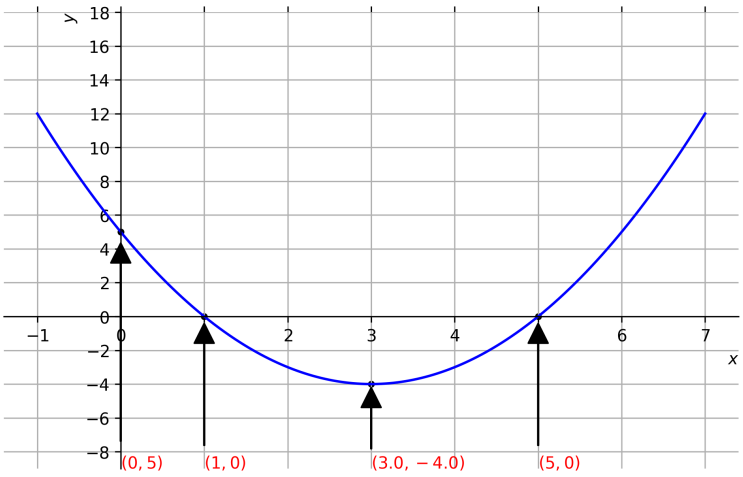
c Find $f^{-1}(x)$. (1 mark)

## Question 7

The parabola below is transformed in the following order:

1. horizontal dilation by a factor of 2
2. vertical dilation by a factor of $\frac{1}{2}$
3. horizontal translation 2 units to the right

Sketch the transformed graph in the following grid.




(3 marks)

## Question 8

Consider the function $f : D \to R$ where the rule is $f(x) = 3\tan(x-4) - 4$.

a State the maximal domain of $f$. (1 mark)

b State the period of $f$. (1 mark)

c State the range of $f$. (1 mark)

d Solve the equation $f(x) = -3\sqrt{3} - 4$ for $x \in \mathbb{R}$. (3 marks)

## Question 9

Consider the functions $f : D \to R$, $f(x) = 12x^2 - 11x - 5$ and $g : D \to R$, $g(x) = e^{-4x}$.

a Find the maximal domain of $f(g(x))$. (1 mark)

b Simplify $g(f(x))$. (1 mark)

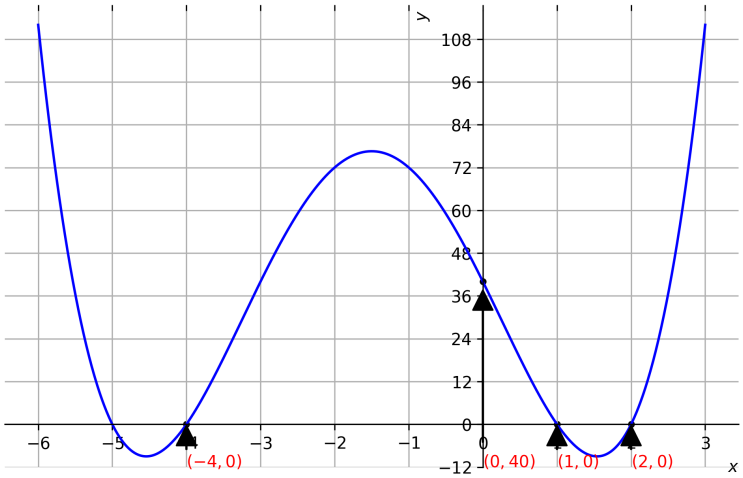
c Solve the equation

$$f(g(x)) = 0$$

(2 marks)

## Solutions

1. (a) C
   (b) A
   (c) B
   (d) D
   (e) C
   (f) A
   (g) B
   (h) D
   (i) B
   (j) C
   (k) B

2. 


3. (a)
$$x^3 - 3x^2 - 16x + 48$$

   (b)
$$3(x-6)(x-5)(x+2)$$

   (c)
$$\mathbb{R}$$

   (d)
$$x^5 - 3x^4 - 16x^3 + 48x^2 - 4x$$

4. (a)

$$\mathbb{R}$$

(b)

$$\left[-\frac{9}{5},\infty\right)$$

(c) $a \geq \frac{7}{10}$

(d)

$$\left[-\frac{9}{5},\infty\right)$$

5. (a)

$$\mathbb{R}$$

(b)

$$\left[-\frac{13}{4},\infty\right)$$

(c)

$$\left[\frac{-\sqrt{12a^2-2a+1}-1}{2a},\ \frac{\sqrt{12a^2-2a+1}-1}{2a}\right]$$

(d)

$$\frac{-3a-\sqrt{21a^2+14a+1}-1}{2a}$$

and

$$\frac{-3a+\sqrt{21a^2+14a+1}-1}{2a}$$

6. (a)

$$\left[\frac{4}{5},\infty\right)$$

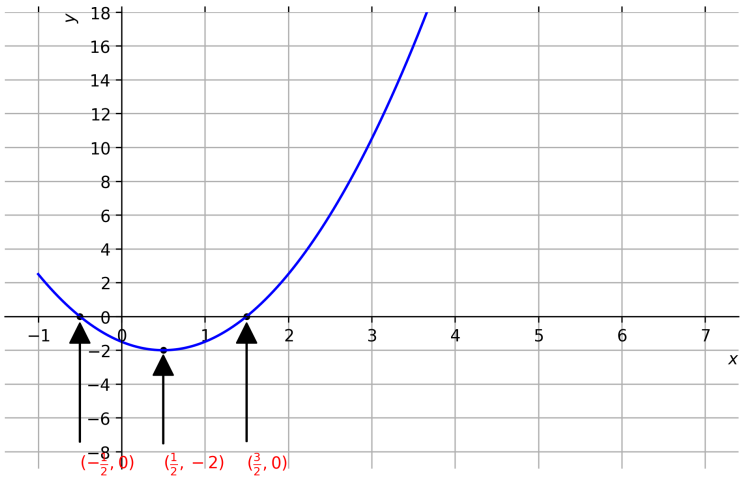
(b)

$$[0,\infty)$$

(c)

$$\frac{x^2}{5}+\frac{4}{5}$$

7.




8. (a) $\mathbb{R} \setminus \left\{\pi n + \frac{\pi}{2} + 4 : n \in \mathbb{Z}\right\}$

(b) $\pi$

(c) $\mathbb{R}$

(d) $x = \pi n - \frac{\pi}{3} + 4,\ n \in \mathbb{Z}$

9. (a)

$$\mathbb{R}$$

(b)

$$e^{-4(3x+1)(4x-5)}$$

(c) There are no real solutions.