MATHMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	**9**	**45**

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(11 marks)

a If

$$f(x) = -3 + \frac{1}{x+4}$$

and

$$g(x) = -x - 5$$

let

$$h(x) = f(g(x))$$

. What is the domain of $h(x)$?

A

$$(-\infty, 1) \cup (1, \infty)$$

B

$$(-\infty, -3) \cup (-3, \infty)$$

C

$$\mathbb{R}$$

D

$$(-\infty, -1) \cup (-1, \infty)$$

(1 mark)

b If

$$f(x) = -5 + \frac{1}{x-4}$$

and

$$g(x) = 3x + 4$$

let

$$h(x) = f(g(x))$$

What are the domain and range of $h(x)$?

A

$$\text{Domain: } (-\infty, -5) \cup (-5, \infty), \text{ Range: } (-\infty, 0) \cup (0, \infty)$$

B

$$\text{Domain: } (-\infty, 0) \cup (0, \infty), \text{ Range: } (-\infty, -5) \cup (-5, \infty)$$

C

$$\text{Domain: } \mathbb{R}, \text{ Range: } (-\infty, -5) \cup (-5, \infty)$$

(1 mark)

c If $f(x)$ is defined as

$$f(x) = 4x - 3$$

and $g(x)$ is defined as

$$g(x) = 2x^2 + x - 4$$

simplify $g(f(x))$.

A

$$2x^2 + 5x - 7$$

B

$$32x^2 - 44x + 11$$

C

$$26x^2 + 13x - 52$$

D

$$2x^2 - 3x - 1$$

(1 mark)

d If $f(x)$ is defined as

$$f(x) = x + 2$$

and $g(x)$ is defined as

$$g(x) = 2x - 5$$

solve the equation

$$g(f(x)) + f(g(x)) = 0$$

A

$$\frac{8}{25}$$

B

$$\frac{19}{20}$$

C

$$1$$

D

$$\frac{25}{61}$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = 3 + \frac{1}{3x + 2}$$

what is the maximal domain of $f^{-1}(x)$?

A

$$\mathbb{R}$$

B

$$\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \infty\right)$$

C

$$(3, \infty)$$

D

$$(-\infty, 3) \cup (3, \infty)$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = \sqrt{x + 3}$$

find $f^{-1}(x)$.

A

$$\left[\frac{x^2}{169} - 3\right]$$

B

$$\left[400x^2 - 3\right]$$

C

$$\left[(x-16)^2-3\right]$$

D

$$\left[x^2-3\right]$$

(1 mark)

g If $f(x)$ is defined as

$$f(x)=\sqrt[3]{5x-2}$$

find $f^{-1}(x)$.

A

$$\left[\frac{2}{5}-\frac{(x-13)^3}{5}\right]$$

B

$$\left[\frac{1728x^3}{5}+\frac{2}{5}\right]$$

C

$$\left[\frac{x^3}{29160}+\frac{2}{5}\right]$$

D

$$\left[\frac{x^3}{5}+\frac{2}{5}\right]$$

(1 mark)

h If $f(x)$ is defined as

$$f(x)=\sqrt{2x+8}+3$$

what is the maximal range of $f^{-1}(x)$?

A

$$[3,\infty)$$

B

$$(-\infty,3]$$

C

$$\mathbb{R}$$

D

$$[-4,\infty)$$

(1 mark)

i A line is parallel to the graph

$$-7x-3y-10=0$$

and passes through the point $(0,6)$. What is the equation of that line?

A

$$y=6-\frac{7x}{3}$$

B

$$y=-7x-3y-10$$

C

$$y=48-\frac{56x}{3}$$

D

$$y = \frac{7x}{3} - 3$$

(1 mark)

j A line is perpendicular to the graph

$$-4x - 10y + 8 = 0$$

and passes through the point $(1, 8.5)$. What is the equation of that line?

A

$$y = \frac{5x}{2} + 6$$

B

$$y = \frac{25x}{2} + 30$$

C

$$y = -\frac{5x}{2} - 3$$

D

$$y = -4x - 10y + 8$$

(1 mark)

k For the following two functions,

$$f(x) = 2ax^2 + ax - 5$$

and

$$g(x) = 2ax + 4$$

what values of a will ensure that the graphs of these functions never intersect?

A

$$a \geq -17, a < -17$$

B

$$a > -72, a < 0$$

C

$$a < -9, a > 4$$

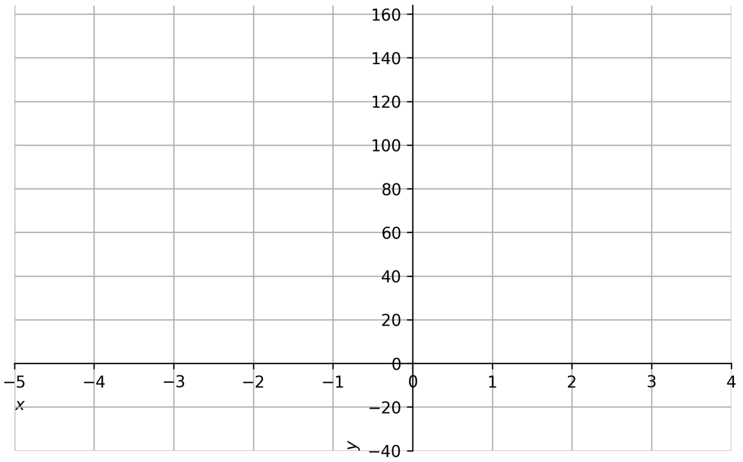
D

$$a \geq -9, a \leq -8$$

(1 mark)

Question 2

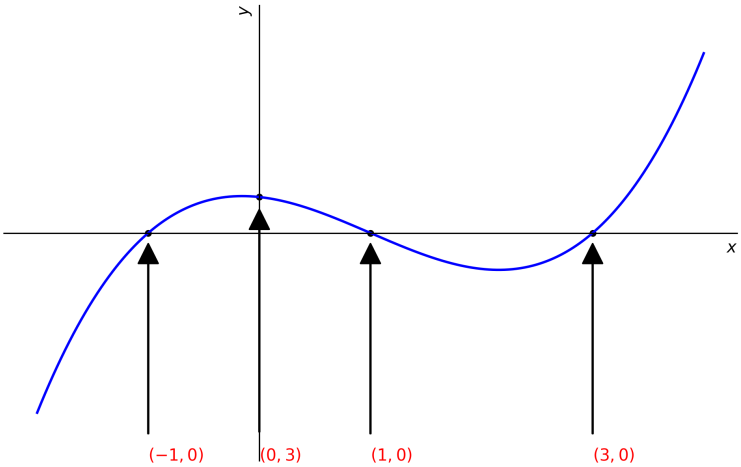
Let $f : R \to R$, $f(x) = x^4 + 2x^3 - 11x^2 - 12x$.
Sketch the graph of $f(x)$ on the following grid.



(4 marks)

Question 3

Consider the following graph.



a Write the rule for the graph. (1 mark)

b This function is transformed according to the rule

$$g(x) = -3f(x-4)$$

Simplify $g(x)$ in terms of x. (2 marks) c Find the maximal domain of $f(x)$. (1 mark)

d Another function $g(x)$ is defined as

$$g(x) = f(x)x^3 + 5x$$

Simplify $g(x)$ in terms of x. (1 mark)

Question 4

Consider the function $f(x) = 25x^2 - 1$.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

c If another function $g(x)$ is defined as

$$g(x) = x - 3$$

solve the equation

$$g(f(x)) = 0$$

(2 marks)

Question 5

Consider the function $f(x) = -4ax^2 + 3a + 3x$.

a What is the domain of $f(x)$ if $a = 1$? (1 mark)

b What is the range of $f(x)$ if $a = -8$? (1 mark)

c This parabola interset with the line $y = a(1 - 3x)$. Find the value(s) of x at which the two graphs intersect. (3 marks)

Question 6

Consider the function $f : D \to R$ where the rule is $f(x) = \sqrt{-3x - 3}$. D is the maximal domain of the function.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

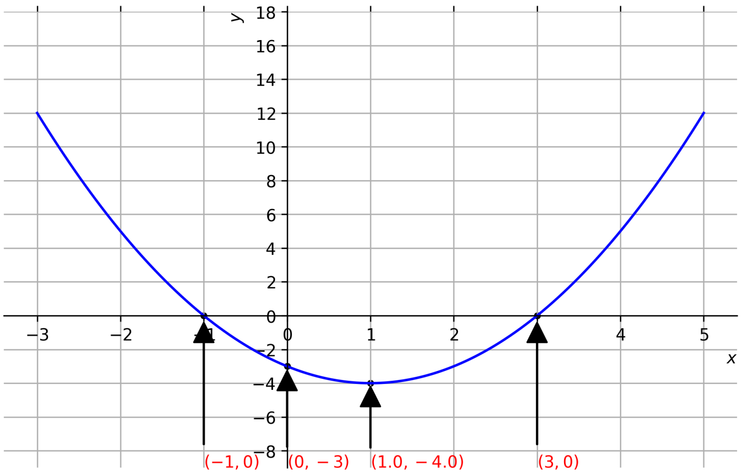
c Find $f^{-1}(x)$. (1 mark)

Question 7

The parabola below is transformed in the following order:

1. vertical dilation by a factor of $\frac{1}{2}$
2. horizontal dilation by a factor of $\frac{1}{2}$
3. horizontal translation 2 units to the right

Sketch the transformed graph in the following grid.



(3 marks)

Question 8

Consider the function $f : D \to R$ where the rule is $f(x) = 4\tan(5x+4) - 1$.

a State the maximal domain of f. (1 mark)

b State the period of f. (1 mark)

c State the range of f. (1 mark)

d Solve the equation $f(x) = -\frac{4\sqrt{3}}{3} - 1$ for $x \in \mathbb{R}$. (3 marks)

Question 9

Consider the functions $f : D \to R$, $f(x) = 12x^2 - 16x + 4$ and $g : D \to R$, $g(x) = e^x$.

a Find the maximal domain of $f(g(x))$. (1 mark)

b Simplify $g(f(x))$. (1 mark)

c Solve the equation

$$f(g(x)) = 0$$

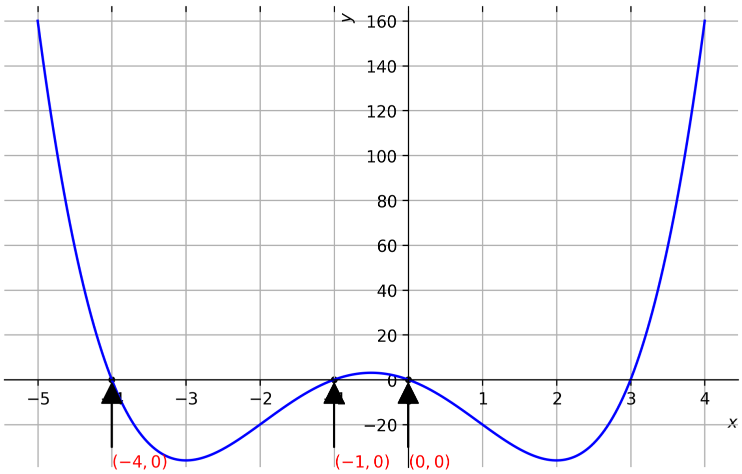
(2 marks)

Solutions

1. (a) D
   (b) B
   (c) B
   (d) C
   (e) D
   (f) D
   (g) D
   (h) D
   (i) A
   (j) A

(k) B

2.



3. (a)

$$x^3 - 3x^2 - x + 3$$

(b)

$$-3(x-7)(x-5)(x-3)$$

(c)

$$\mathbb{R}$$

(d)

$$x^6 - 3x^5 - x^4 + 3x^3 + 5x$$

4. (a)

$$\mathbb{R}$$

(b)

$$[-1, \infty)$$

(c)

$$\left[-\frac{2}{5}, \frac{2}{5}\right]$$

5. (a)

$$\mathbb{R}$$

(b)

$$\left[-\frac{3081}{128}, \infty\right)$$

(c)

$$\frac{3a - \sqrt{41a^2 + 18a + 9} + 3}{8a}$$

and

$$\frac{3a + \sqrt{41a^2 + 18a + 9} + 3}{8a}$$

6. (a)

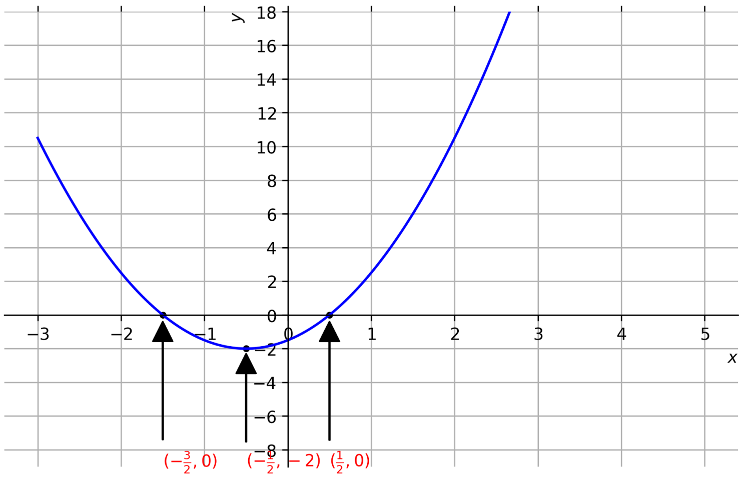
$$(-\infty, -1]$$

(b)

$$[0, \infty)$$

(c)

$$-\frac{x^2}{3} - 1$$

7. 

8. (a)

$$\mathbb{R} \setminus \left\{ \frac{\pi n}{5} - \frac{4}{5} + \frac{\pi}{10} : n \in \mathbb{Z} \right\}$$

(b) $\frac{\pi}{5}$

(c) $\mathbb{R}$

(d) $x = \frac{\pi n}{5} - \frac{4}{5} - \frac{\pi}{30}$, $n \in \mathbb{Z}$

9. (a)

$$\mathbb{R}$$

(b)

$$e^{4(x-1)(3x-1)}$$

(c)

$$-\log(3)$$

and

$$0$$