

MATHEMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	37

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1**(8 marks)**a If $f(x)$ is defined as

$$f(x) = 4x - 3$$

and $g(x)$ is defined as

$$g(x) = x^2 - 3x - 5$$

use the chain rule to find $g'(f(x))$.

A

$$x^2 - 7x - 2$$

B

$$x^2 + 29x - 41$$

C

$$608x - 684$$

D

$$32x - 36$$

(1 mark)b If $f(x)$ is defined as

$$f(x) = -5x^2 + 4x - 4$$

and $g(x)$ is defined as

$$g(x) = -3x^2 - 4x + 5$$

use the chain rule to find $g'(f(x))$.

A

$$-2400x^3 + 2880x^2 - 2368x + 640$$

B

$$2x^2 - 8x + 9$$

C

$$-300x^3 + 357x^2 - 300x + 85$$

D

$$-300x^3 + 360x^2 - 296x + 80$$

(1 mark)c Differentiate the following expression with respect to z :

$$(-4z - 2)^{\frac{6}{7}}$$

A

$$\frac{3\sqrt[7]{2}}{28(-2z - 1)^{\frac{13}{7}}}$$

B

$$-3z^2 + 5z - 5$$

C

$$\frac{48\sqrt[7]{2}}{7(-2z - 1)^{\frac{13}{7}}}$$

D

$$\frac{6\sqrt[7]{2}}{7(-2z-1)^{\frac{13}{7}}}$$

(1 mark)

d Differentiate the following expression with respect to x

$$\frac{-5x-3}{\log(-3x^2+4x-1)}$$

A

$$\frac{2(3x-2)(5x+3)+5(-3x^2+4x-1)\log(-3x^2+4x-1)}{(3x^2-4x+1)\log(-3x^2+4x-1)^2}$$

B

$$-2x^3+x^2-3x+4$$

C

$$\frac{9(2(3x-2)(5x+3)+5(-3x^2+4x-1)\log(-3x^2+4x-1))}{(3x^2-4x+1)\log(-3x^2+4x-1)^2}$$

D

$$\frac{2(3x-2)(5x+3)+5(-3x^2+4x-1)\log(-3x^2+4x-1)}{5(3x^2-4x+1)\log(-3x^2+4x-1)^2}$$

(1 mark)

e Find the expression for

$$\frac{dy}{dx}$$

for the following:

$$\frac{y(-17x-81)}{81} = \frac{85x^3}{81} + \frac{118x^2}{27} - \frac{328x}{81} - 5$$

A

$$\frac{3}{10} - x$$

B

$$15 - 50x$$

C

$$-3x^3 + x^2 - x - 4$$

D

$$3 - 10x$$

(1 mark)

f Differentiate the polynomial with respect to y

$$\frac{-4y^7 - y^6 - 4y^5 + 2y^4 + 4y^3 + 5y^2 - y - 5}{y^{10}}$$

A

$$2y^7 + 3y^6 + y^5 + 2y^4 + 5y^3 - 2y^2 - 2y - 5$$

B

$$\frac{12y^7 + 4y^6 + 20y^5 - 12y^4 - 28y^3 - 40y^2 + 9y + 50}{10y^{11}}$$

C

$$\frac{12y^7 + 4y^6 + 20y^5 - 12y^4 - 28y^3 - 40y^2 + 9y + 50}{y^{11}}$$

D

$$\frac{5(12y^7 + 4y^6 + 20y^5 - 12y^4 - 28y^3 - 40y^2 + 9y + 50)}{y^{11}}$$

(1 mark)

g Differentiate the following expression with respect to c

$$\frac{1 - 3c}{\sqrt{2c^2 - 4c + 3}}$$

A

$$\frac{4c - 7}{7(2c^2 - 4c + 3)^{\frac{3}{2}}}$$

B

$$\frac{15(4c - 7)}{(2c^2 - 4c + 3)^{\frac{3}{2}}}$$

C

$$\frac{4c - 7}{(2c^2 - 4c + 3)^{\frac{3}{2}}}$$

D

$$2c^3 + 3c^2 - 4c - 3$$

(1 mark)

h Differentiate the following expression with respect to b

$$e^{9m} \cos^2(2b^2 m^2)$$

A

$$-\frac{8bm^2 e^{9m} \sin(2b^2 m^2) \cos(2b^2 m^2)}{15}$$

B

$$-8bm^2 e^{9m} \sin(2b^2 m^2) \cos(2b^2 m^2)$$

C

$$-56bm^2 e^{9m} \sin(2b^2 m^2) \cos(2b^2 m^2)$$

D

$$-12e^{-9m} \sin^2(3m)$$

(1 mark)

Question 2

A festival program has 11 acts. A running order of 8 different acts is chosen. Two of the acts are the band act and the poetry act.

a Using permutations, how many different running orders are possible?

(1 mark)

- b How many running orders include both the band act and the poetry act, with the band act before the poetry act and exactly one act between them? (3 marks)

- c How many running orders place both the band act and the poetry act in odd-numbered positions? (3 marks)

- d How many running orders exclude both the band act and the poetry act? (2 marks)

Question 3

A shelf contains 5 science books and 3 history books. Two books are selected one after the other without replacement. Let event A be "the first book is a science book" and event B be "the second book is a history book".

- a Find $P(A)$ and $P(B)$. (1 mark)

- b Find $P(A \cap B)$. (1 mark)

- c Determine whether events A and B are independent or dependent. Justify your answer. (2 marks)

- d Find the probability that neither A nor B occurs. (2 marks)

Question 4

From a collection of 90 cards, one card is chosen at random. Each card is white or purple and is marked with the letter D , E , or F . The white-card counts are 12, 14, and 10 for D , E , and F , while the purple-card counts are 18, 18, and 18. Let X denote the letter on the selected card.

- a What is the probability that the card drawn is purple and labelled F ? (1 mark)

- b Find the probability distribution of X . (2 marks)

- c What is the probability that $X \neq D$? (1 mark)

- d What is the probability that the card is purple given that $X = F$? (2 marks)

Question 5

Four colours of marbles are mixed in a box: red, yellow, blue, and white. Red, yellow, and white are equally likely to be drawn, while blue is 2 times as likely as white. One marble is selected at random.

- a Let the probability of drawing a white marble be x . Express the probabilities of drawing the other colours in terms of x . (1 mark)

- b Use the fact that the total probability is 1 to find the value of x and hence determine the probability of each colour. (2 marks)

- c Using combinations, how many different pairs of colours can be chosen from red, yellow, blue, and white? (1 mark)

- d What is the probability of drawing a marble that is not blue? (1 mark)

Question 6

A continuous random variable X has a probability density function

$$p(x) = \frac{e^x}{6}, \log_e(3) \leq x \leq \log_e(9)$$

and zero otherwise.

(3 marks)

- a Evaluate the mean.

(1 mark)

b Evaluate $\Pr(\log_e(3) \leq X \leq \log_e(8))$.

(1 mark)

c Evaluate $\Pr(X < \log_e(8))$.

(1 mark)

Solutions

- D
 - D
 - D
 - A
 - D
 - C
 - C
 - B
- Choose and arrange 8 acts from the 11 available acts. Therefore ${}^{11}P_8 = 6652800$.
 - Choose the starting position of the band act so there is exactly one act between the two named acts. Then fill the remaining positions from the other acts. Therefore $6 \times {}^9P_6 = 362880$.
 - There are four odd-numbered positions in the running order. Choose and order two of them for the named acts, then fill the other positions from the remaining acts. Therefore ${}^4P_2 \times {}^9P_6 = 725760$.
 - Exclude the two named acts, then choose and arrange the full running order from the remaining acts. Therefore ${}^9P_8 = 362880$.
- $P(A) = 0.625$, $P(B) = 0.375$
 - $\frac{15}{56}$
 - Dependent, since $P(A \cap B) = 0.267857 \neq P(A)P(B) = 0.234375$
 - $\frac{15}{56}$
- $\frac{1}{5}$
 - $P(X = D) = \frac{1}{3}$, $P(X = E) = 0.355556$, $P(X = F) = 0.311111$
 - $\frac{2}{3}$
 - $\frac{9}{14}$
- $P(\text{red}) = x$, $P(\text{yellow}) = x$, $P(\text{blue}) = 2x$

(b) $x = 0.2$, $P(\text{red}) = 0.2$, $P(\text{yellow}) = 0.2$, $P(\text{blue}) = 0.4$, $P(\text{white}) = 0.2$

(c) 6

(d) $\frac{3}{5}$

6. (a)

$$-1 + \frac{5 \log(3)}{2}$$

(b)

$$\frac{5}{6}$$

(c)

$$\frac{5}{6}$$