

MATHEMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	36

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1**(8 marks)**a If $f(x)$ is defined as

$$f(x) = 3x - 2$$

and $g(x)$ is defined as

$$g(x) = 5x^2 + 5x + 5$$

use the chain rule to find $g'(f(x))$.

A

$$1350x - 675$$

B

$$5x^2 + 95x - 40$$

C

$$90x - 45$$

D

$$5x^2 + 2x + 7$$

(1 mark)b If $f(x)$ is defined as

$$f(x) = -5x^2 + 4x - 5$$

and $g(x)$ is defined as

$$g(x) = -3x^2 + x - 5$$

use the chain rule to find $g'(f(x))$.

A

$$-300x^3 + 357x^2 - 405x + 119$$

B

$$-2400x^3 + 2880x^2 - 3248x + 992$$

C

$$-300x^3 + 360x^2 - 406x + 124$$

D

$$2x^2 - 3x$$

(1 mark)c Differentiate the following expression with respect to d :

$$(-5d^3 + d^2 + 4d - 2)^{\frac{8}{7}}$$

A

$$-2d^2 + 3d - 5$$

B

$$\frac{16(15d^2 - 2d - 4)}{(-5d^3 + d^2 + 4d - 2)^{\frac{15}{7}}}$$

C

$$\frac{8(15d^2 - 2d - 4)}{7(-5d^3 + d^2 + 4d - 2)^{\frac{15}{7}}}$$

D

$$\frac{8(15d^2 - 2d - 4)}{119(-5d^3 + d^2 + 4d - 2)^{\frac{15}{7}}}$$

(1 mark)

d Differentiate the following expression with respect to l

$$\frac{l - 5}{\log(-3l^2 + 2l - 3)}$$

A

$$-3l^3 - l^2 + 3l + 5$$

B

$$\frac{-2(l - 5)(3l - 1) + (3l^2 - 2l + 3) \log(-3l^2 + 2l - 3)}{(3l^2 - 2l + 3) \log(-3l^2 + 2l - 3)^2}$$

C

$$\frac{-2(l - 5)(3l - 1) + (3l^2 - 2l + 3) \log(-3l^2 + 2l - 3)}{9(3l^2 - 2l + 3) \log(-3l^2 + 2l - 3)^2}$$

D

$$\frac{6(-2(l - 5)(3l - 1) + (3l^2 - 2l + 3) \log(-3l^2 + 2l - 3))}{(3l^2 - 2l + 3) \log(-3l^2 + 2l - 3)^2}$$

(1 mark)

e Find the expression for

$$\frac{dy}{dx}$$

for the following:

$$\frac{y(-x - 2)}{2} = \frac{3x^3}{2} + 5x^2 + \frac{7x}{2} - 1$$

A

$$-4x^3 + 4x^2 + 2x + 5$$

B

$$-\frac{3x}{5} - \frac{2}{5}$$

C

$$-30x - 20$$

D

$$-6x - 4$$

(1 mark)

f Differentiate the polynomial with respect to c

$$\frac{2c^8 + 4c^7 + c^6 - 5c^5 + 2c^4 - 4c^3 - 4c^2 - 2c + 5}{c^{14}}$$

A

$$\frac{-12c^8 - 28c^7 - 8c^6 + 45c^5 - 20c^4 + 44c^3 + 48c^2 + 26c - 70}{10c^{15}}$$

B

$$3c^8 + 5c^7 - 5c^6 + 2c^5 + 3c^4 - 3c^3 - c^2 + 5c - 2$$

C

$$\frac{-12c^8 - 28c^7 - 8c^6 + 45c^5 - 20c^4 + 44c^3 + 48c^2 + 26c - 70}{c^{15}}$$

D

$$\frac{5(-12c^8 - 28c^7 - 8c^6 + 45c^5 - 20c^4 + 44c^3 + 48c^2 + 26c - 70)}{c^{15}}$$

(1 mark)

g Differentiate the following expression with respect to b

$$\frac{4(-b-1)}{\sqrt{2b^2-5b+1}}$$

A

$$4b^3 - 4b^2 - 2b - 5$$

B

$$\frac{9b-7}{5(2b^2-5b+1)^{\frac{3}{2}}}$$

C

$$\frac{2(9b-7)}{(2b^2-5b+1)^{\frac{3}{2}}}$$

D

$$\frac{12(9b-7)}{(2b^2-5b+1)^{\frac{3}{2}}}$$

(1 mark)

h Differentiate the following expression with respect to z

$$-15e^{3t^2} \cos(z^2)$$

A

$$30ze^{3t^2} \sin(z^2)$$

B

$$\frac{10ze^{3t^2} \sin(z^2)}{3}$$

C

$$-4e^{4tz^2} \sin^2(3tz^2)$$

D

$$420ze^{3t^2} \sin(z^2)$$

(1 mark)

Question 2

A leadership team is formed by assigning the five distinct roles of chair, deputy chair, secretary, treasurer, and communications officer from 10 students. Three of the students are Raneem, Leroy, and Antonio.

a Using permutations, how many different leadership teams can be formed? (1 mark)

- b How many leadership teams include all of Raneem, Leroy, and Antonio, with Raneem as chair and Leroy in a role before Antonio? (3 marks)

- c How many leadership teams contain exactly two of Raneem, Leroy, and Antonio, with neither selected student as treasurer? (3 marks)

- d How many leadership teams have Raneem as chair and Leroy as treasurer, with Antonio not selected? (2 marks)

Question 3

A shelf contains 7 science books and 3 history books. Two books are selected one after the other without replacement. Let event A be "the first book is a science book" and event B be "the second book is a history book".

- a Find $P(A)$ and $P(B)$. (1 mark)

- b Find $P(A \cap B)$. (1 mark)

- c Determine whether events A and B are independent or dependent. Justify your answer. (2 marks)

d Find $P(B | A)$.

(1 mark)

Question 4

From a collection of 80 cards, one card is chosen at random. Each card is white or purple and is marked with the letter D , E , or F . The white-card counts are 10, 12, and 8 for D , E , and F , while the purple-card counts are 15, 18, and 17. Let X denote the letter on the selected card.

a What is the probability that the card drawn is purple and labelled F ?

(1 mark)

b Find the probability distribution of X .

(2 marks)

c What is the probability that $X = E$?

(1 mark)

d What is the probability that the card is purple given that $X = F$?

(2 marks)

Question 5

A box contains red, yellow, blue, and white marbles. The probabilities of drawing a red marble, a yellow marble, and a white marble are equal, and the probability of drawing a blue marble is 3 times the probability of drawing a white marble. One marble is drawn at random.

- a Let the probability of drawing a white marble be x . Express the probabilities of drawing the other colours in terms of x . (1 mark)

- b Use the fact that the total probability is 1 to find the value of x and hence determine the probability of each colour. (2 marks)

- c Using combinations, how many different pairs of colours can be chosen from red, yellow, blue, and white? (1 mark)

- d What is the probability of drawing a marble that is not yellow? (1 mark)

Question 6

A continuous random variable X has a probability density function

$$p(x) = \frac{4}{3} - \frac{2x}{9}, 3 \leq x \leq 6$$

and zero otherwise.

(3 marks)

- a Evaluate the mean.

(1 mark)

b Evaluate $\Pr(-5 \leq X \leq 8)$.

(1 mark)

c Evaluate $\Pr(X < 3)$.

(1 mark)

Solutions

1. (a) C
(b) C
(c) C
(d) B
(e) D
(f) C
(g) C
(h) A
2. (a) Assign the five distinct roles from the 10 students. Therefore $^{10}P_5 = 30240$.
(b) Fix Raneem as chair. Choose two of the remaining four roles for Leroy and Antonio, with the earlier chosen role assigned to Leroy. Then fill the last two roles from the other students. Therefore $\binom{4}{2} \times {}^7P_2 = 252$.
(c) Choose which two of the three named students are selected. Assign them to two of the four allowed non-treasurer roles, then fill the remaining three roles from the other students. Therefore $3 \times {}^4P_2 \times {}^7P_3 = 7560$.
(d) Fix Raneem as chair and Leroy as treasurer, exclude Antonio, then assign the remaining three roles from the other students. Therefore ${}^7P_3 = 210$.
3. (a) $P(A) = 0.7$, $P(B) = 0.3$
(b) $\frac{7}{30}$
(c) Dependent, since $P(A \cap B) = 0.233333 \neq P(A)P(B) = 0.21$
(d) $\frac{1}{3}$
4. (a) $\frac{17}{80}$
(b) $P(X = D) = 0.3125$, $P(X = E) = 0.375$, $P(X = F) = 0.3125$
(c) $\frac{3}{8}$
(d) $\frac{17}{25}$
5. (a) $P(\text{red}) = x$, $P(\text{yellow}) = x$, $P(\text{blue}) = 3x$

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- (b) $x = \frac{1}{6}, P(\text{red}) = \frac{1}{6}, P(\text{yellow}) = \frac{1}{6}, P(\text{blue}) = 0.5, P(\text{white}) = \frac{1}{6}$
- (c) 6
- (d) $\frac{5}{6}$
6. (a) 4
- (b) 1
- (c) 0