MATHMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	**5**	**28**

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(4 marks)

a If $f(x)$ is defined as

$$f(x) = \frac{1}{-2x-4}$$

$g(x)$ is defined as

$$g(x) = 4x^2 + x - 3$$

and $h(x)$ is defined as

$$h(x) = -3x - 1$$

use the chain rule to find $g^{'}(f(h(x)))$.

A

$$4x^2 + x - 3 - \frac{48}{216x^3 - 216x^2 + 72x - 8} - \frac{6}{36x^2 - 24x + 4}$$

B

$$4x^2 + x - 3 - \frac{1}{-2x-4}$$

C

$$\frac{9(-x-1)}{2(3x-1)^3}$$

D

$$-\frac{144}{216x^3 - 216x^2 + 72x - 8} - \frac{18}{36x^2 - 24x + 4}$$

(1 mark)

b Differentiate the following expression with respect to d

$$-2e^{10d^2}\tan\left(4ad^2\right)$$

A

$$-5e^{-8ad^2}\tan^2\left(a^2\right)$$

B

$$\frac{8d\left(-\frac{2a}{\cos^2\left(4ad^2\right)} - 5\tan\left(4ad^2\right)\right)e^{10d^2}}{7}$$

C

$$8d\left(-\frac{2a}{\cos^2\left(4ad^2\right)} - 5\tan\left(4ad^2\right)\right)e^{10d^2}$$

D

$$48d\left(-\frac{2a}{\cos^2\left(4ad^2\right)} - 5\tan\left(4ad^2\right)\right)e^{10d^2}$$

(1 mark)

c Differentiate the following expression with respect to m

$$6b^2m^3\cos(m) + 2\sin^2\left(m^3\right)$$

A

$$84m^2\left(-b^2m\sin(m) + 3b^2\cos(m) + \sin\left(2m^3\right)\right)$$

B

$$\frac{2m^2\left(-b^2m\sin(m) + 3b^2\cos(m) + \sin\left(2m^3\right)\right)}{5}$$

C

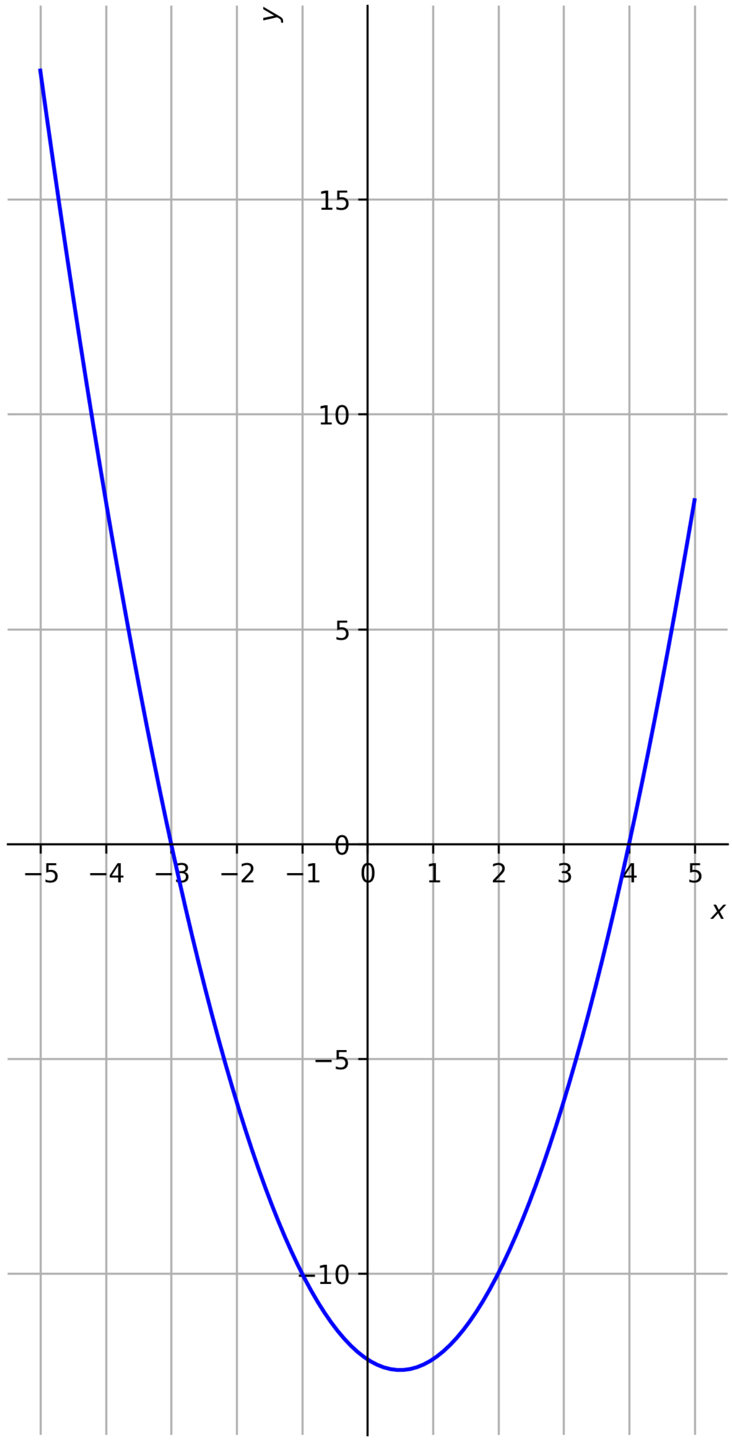
$$6m^2\left(-b^2m\sin\left(m\right)+3b^2\cos\left(m\right)+\sin\left(2m^3\right)\right)$$

D

$$\tan\left(3m^2\right)$$

(1 mark)

d What is the sign of the tanget at point 1 in the following graph? (1 mark)



Question 2

A rescue flare is launched and its height above the deck, in metres, t seconds after launch is modelled by:

$$h(t) = -t^2 + 6t - 8.$$

where $t \geq 0$.

a Differentiate $h(t)$ to find the vertical velocity. (1 mark)

b Determine the time at which the flare reaches its maximum height. (1 mark)

c Calculate the maximum height of the flare. (1 mark)

d When does the flare return to deck level? (1 mark)

Question 3

A spring-mounted platform moves horizontally from its equilibrium position according to

$$s(t) = \frac{2}{5}\sin(3t),$$

where $s(t)$ is the displacement in metres and t is the time in seconds.

a Differentiate $s(t)$ to find the velocity function $v(t)$. (1 mark) b Find the velocity of the platform at $t = \dfrac{\pi}{9}$ seconds. (2 marks)

c Determine the first positive time the platform passes through its equilibrium position. (2 marks)

d Find the maximum speed of the platform and the maximum magnitude of its acceleration.(2 marks)

Question 4

The height of a marina pontoon above its reference level is modelled by

$$h(t) = 3 - \frac{3}{2}\sin\left(\frac{\pi t}{5}\right),$$

where $h(t)$ is measured in metres and t is the number of hours after 6 am.

a Differentiate $h(t)$ to find the rate at which the pontoon height is changing. (1 mark)

b Find the rate of change of the pontoon height at $t = 1$ hour and state whether the pontoon is rising or falling. (2 marks)

c Determine the first time after 6 am that the pontoon reaches its lowest point. (2 marks) d During the first 10 hours, when is the pontoon rising most rapidly, and what is that rate?(2 marks)

Question 5

Suppose

$$f^{'}(x) = 10x^2 - 10x - 20$$

a Solve $f'(x) = 0$ to find the possible x-coordinates of stationary points. (1 mark)

b Differentiate $f'(x)$ to find $f''(x)$. (1 mark)

c State the intervals on which f is increasing and the intervals on which f is decreasing. (2 marks)

d If $(2, -\frac{112}{3})$ is a stationary point, find the expression of $f(x)$. (2 marks)

Solutions

1. (a) C
   (b) C
   (c) C
   (d) positive

2. (a) Differentiate the given function to obtain 6 - 2 t.
   (b) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives 3.
   (c) Find the turning time by setting the derivative equal to 0, then substitute back into the original function. This gives 1.
   (d) Set the model equal to the required value and solve for the variable. This gives 4.

3. (a) Differentiate the given function to obtain $v(t) = \frac{6\cos(3t)}{5}$.
   (b) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives $v\left(\frac{\pi}{9}\right) = \frac{3}{5} \approx 0.6 \quad \text{m s}^{-1}$.
   (c) Using the required working, the answer is $t = \frac{\pi}{3} \approx 1.047 \quad \text{s}$.
   (d) Using the required working, the answer is $v_{\max} = 1.2 \quad \text{m s}^{-1}, \quad \text{maximum acceleration magnitude} = 3.6 \quad \text{m s}^{-2}$.

4. (a) Differentiate the given function to obtain $h'(t) = -\frac{3\pi\cos\left(\frac{\pi t}{5}\right)}{10}$.
   (b) Differentiate first to get the relevant rate function, then substitute the required value if needed. This gives $h'(1) = -\frac{3\pi\left(1+\sqrt{5}\right)}{40} \approx -0.762 \quad \text{m h}^{-1}$, so the pontoon is falling..
   (c) Set the model equal to the required value and solve for the variable. This gives $t = 2.5 \quad \text{hours}$.
   (d) The pontoon rises most rapidly at $t = 5$ hours, with $h'(5) = \frac{3\pi}{10} \approx 0.942 \quad \text{m h}^{-1}$.

5. (a) Set the first derivative equal to 0 to find the stationary point, then classify it using the second derivative or sign changes. This gives $[-1,\ 2]$.
   (b) Differentiate the given function to obtain 20 x - 10.
   (c) Using the required working, the answer is Increasing on $(-\infty,-1)\cup(2,\infty)$, decreasing on $(-1,2)$.
   (d) Integrate the given derivative to obtain f(x), then use the stationary point to determine the constant of integration. This gives

$$f(x) = \frac{10x^3}{3} - 5x^2 - 20x - 4$$

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