Tips & tricks: Exponential equations

1 Outline

This article will provide tips and tricks for solving exponential equations at various levels of difficulty. The following are the key tips to remember whenever you are solving an exponential equation:

- make sure that the basis is the same on both sides of the equation;
- don't be tricked with the negative sign! Remember that the odd power of a negative base is still negative, and the even power of a negative base is positive;
- don't be scared if the final equation cannot be solved algebraicly; it's what we've learned logarithms for!

2 Tricky problems

■ **Example 1** Solve the following equation for x:

$$(-4)^{3x-2} = -64$$

Solution: We want to have the same base on both sides. The base is -4, so it should be as such on the right side as well. But the right side is a bit confusing here:

$$-64 = -(4^3)$$

which means we have $+4$ as a base on the right side, but we have -4 as the base on the left side. What should we do? Well, do you notice that the index in $-(4^3)$ is odd? This means that we can slip the negative sign into the base without harming anyone:

$$-(4^3) = (-4)^3$$

And there you go, you have -4 as the base on both sides. Let's then solve the equation. It's a piece of cake now.

$$\begin{aligned} (-4)^{3x-2} &= -64 \\ (-4)^{3x-2} &= (-4)^3 \\ 3x-2 &= 3 \\ x = \frac{5}{3} & \end{aligned}$$

■ **Example 2** Solve the following equation for x:

$$4\,(-5)^{3-4y} = -\frac{4}{125}$$

Solution: Let's first cross out 4 from both sides, and try to find a common base:

$$\begin{aligned} 4\,(-5)^{3-4y} &= -\frac{4}{125} \\ (-5)^{3-4y} &= -5^{-3} \end{aligned}$$

So again we have the basis on the two sides differing in sign. Like we did before, we make use of the fact that -5^{-3} is a power of an odd number - even though it is a negative odd number, but that doesn't matter. So:

$$\begin{aligned}(-5)^{3-4y} &= -5^{-3}\\(-5)^{3-4y} &= (-5)^{-3}\end{aligned}$$

and now we have the same base on both sides, -5, so the solution follows easily now:

$$\begin{aligned}(-5)^{3-4y} &= (-5)^{-3}\\3-4y &= -3\\y &= \frac{3}{2}\end{aligned} \tag{1}$$

■ **Example 3** Solve the following equation for x:

$$-5\cdot 16^d + 6\cdot 2^{2d} + 2 = 7\cdot 4^d$$

Solution: Just don't freak out because the basis are looking very different. They are not different actually; they share the base of 2, don't they?
In some exponential problems, the equation might reduce into a quadratic equation in terms of the base raised to some index. That's what we will be dealing with here.
So, let's dissect the basis in the equation:

- 16^d can be considered 4^{2d}
- 2^{2d} can be considered 4^d

so what we actually have here are those two quantities: 4^{2d} and 4^d. If we consider that $x = 4^d$, then by the index laws, $x^2 = 4^{2d}$. So indeed, we have a quadratic equation in the variable x:

$$-5x^2 + 6x + 2 = 7x$$

$$5x^2 x - 2 = 0$$

The two solutions on the CAS are: -0.74031 and 0.54031. Obviously, we cannot take account of the negative solution, otherwise we will have to deal with the logarithm of a negative number. So the only solution that we will account for is $x = 0.54031$. Based on this, it is time to find d:

$$\begin{aligned}x &= 0.54031\\4^d &= 0.54031\\d &= -0.44407\end{aligned}$$

3 Exercise

Now let's do some exercise problems on www.mathpractice.xyz.
Exercise 1 Click here: Equations involving exponents.