MATHMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
7	**7**	**35**

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

(3 marks)

a If $f(x)$ is defined as

$$f(x) = \frac{4x-5}{-5x-3}$$

find $f^{-1}(x)$. (1 mark)

b If $f(x)$ is defined as

$$f : \mathbb{R} \to \mathbb{R}, f(x) = 5x^2 + 3x - 5$$

evaluate $f(-8) + f(-5)$. (1 mark)

c The function $f(u)$ is defined as follows:

$$f(u) = \begin{cases} -u & u \leq 6 \\ -6u & u > 6 \end{cases}$$

Find the value of the function at $u = -15$. (1 mark)

Question 2

(7 marks)

a If $f(x)$ is defined as

$$f(x) = -2x - 2$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\mathbb{R}$$

Range:

$$(-\infty, -9]$$

B Domain:

$$\mathbb{R}$$

Range:

$$(-\infty, -16]$$

C Domain:

$$\mathbb{R}$$

Range:

$$(-\infty, \infty)$$

D Domain:

$$\mathbb{R}$$

Range:

$$(-\infty, -13]$$

(1 mark)

b If $f(x)$ is defined as

$$f(x) = -5 + \frac{1}{3x+1}$$

what is the maximal range of $f^{-1}(x)$?

A

$$\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$$

B

$$\left(-\frac{1}{3}, \infty\right)$$

C

$$\mathbb{R}$$

D

$$(-\infty, -5) \cup (-5, \infty)$$

(1 mark)

c If $f(x)$ is defined as

$$f(x) = 2x + 1, x > 10$$

what is the range of $f(x)$?

A

$$(-4, \infty)$$

B

$$(215, \infty)$$

C

$$\left[\frac{15}{8}, \infty\right)$$

D

$$(21, \infty)$$

(1 mark)

d If

$$f(x) = 4 + \frac{1}{x+1}$$

and

$$g(x) = 4x + 1$$

let

$$h(x) = f(g(x))$$

. What is the domain of $h(x)$?

A

$$\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$$

B

$$(-\infty, 4) \cup (4, \infty)$$

C

$$\mathbb{R}$$

D

$$\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = 3 - 2x$$

and $g(x)$ is defined as

$$g(x) = 2x - 1$$

solve the equation

$$g(f(x)) + f(g(x)) = 2x$$

A

$$\frac{17}{50}$$

B

$$\frac{22}{25}$$

C

$$\frac{47}{54}$$

D

$$1$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = \sqrt[3]{1 - 4x}$$

find $f^{-1}(x)$.

A

$$\left[\frac{1}{4} - \frac{x^3}{4}\right]$$

B

$$\left[\frac{1}{4} - \frac{x^3}{500}\right]$$

C

$$\left[\frac{1}{4} - 2x^3\right]$$

D

$$\left[\frac{(x-10)^3}{4} + \frac{1}{4}\right]$$

(1 mark)

g If $f(x)$ is defined as

$$f(x) = \sqrt[4]{3x-2}$$

find $f^{-1}(x)$.

A

$$\left[\frac{16x^4}{3} + \frac{2}{3}\right]$$

B

$$\left[\frac{(x-9)^4}{3} + \frac{2}{3}\right]$$

C

$$\left[\frac{x^4}{3} + \frac{2}{3}\right]$$

D

$$\left[\frac{x^4}{768} + \frac{2}{3}\right]$$

(1 mark)

Question 3

Let $f(x) = 2^x + k$ and $g(x) = k + x$, where k is a constant.

a State the domain of f. (1 mark)

b State the range of f. (1 mark)

c Find the inverse function $f^{-1}(x)$ in terms of k. (2 marks)

d Given that $f^{-1}(11) = 3$, find k. (2 marks)

Question 4

Let $f : D = \{x \in \mathbb{R} : x \geq p\} \to \mathbb{R}$ be defined by $f(x) = x^2 + 2x - 2$, where p is a constant.

a State the maximal domain of the rule for f before the restriction is applied. (1 mark)

b Find the value of p such that f is one-to-one on D and therefore has an inverse.(2 marks)

c Hence state the range of f on this restricted domain. (1 mark)

d Find the inverse function $f^{-1}(x)$. (2 marks)

Question 5

Consider the function $f : D \to R$ where the rule is $f(x) = 3 - \sin(3x - 1)$.

a State the domain of f. (1 mark)

b State the amplitude and period of f. (1 mark)

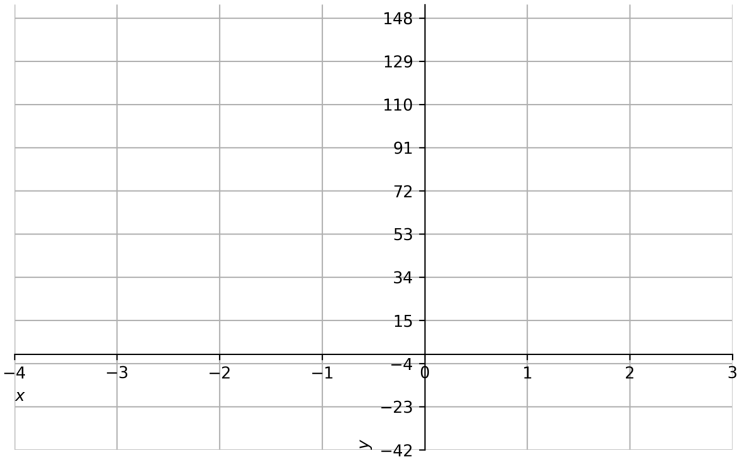
c Find the maximum and minimum values of f. (1 mark)

d Solve the equation $f(x) = \frac{5}{2}$ for $x \in \mathbb{R}$. (3 marks)

Question 6

Let $f : R \to R$, $f(x) = x^4 - x^3 - 7x^2 + 13x - 6$.

Sketch the graph of $f(x)$ on the following grid.



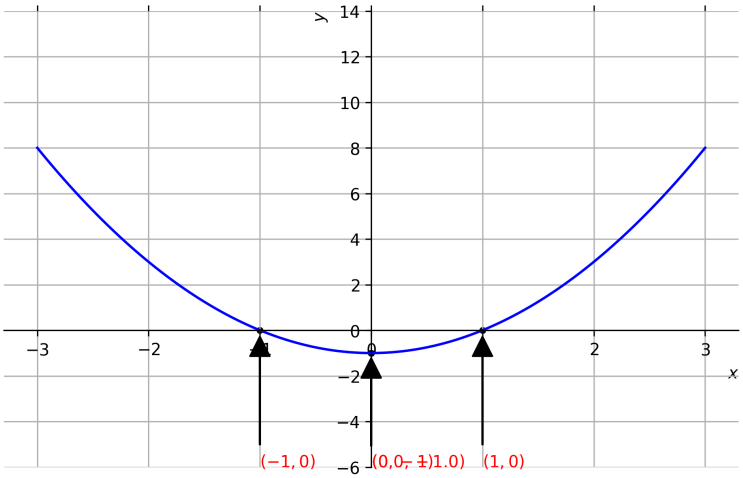
(4 marks)

Question 7

The parabola below is transformed in the following order:

1. horizontal dilation by a factor of 2
2. vertical dilation by a factor of $\frac{1}{3}$
3. horizontal translation 1 units to the right

Sketch the transformed graph in the following grid.



(3 marks)

Solutions

1. (a)

$$\left[\frac{5-3x}{5x+4}\right]$$

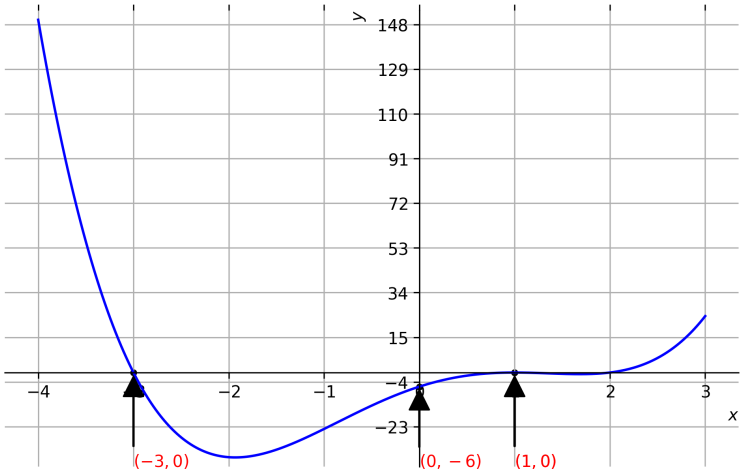
(b) 396

(c)

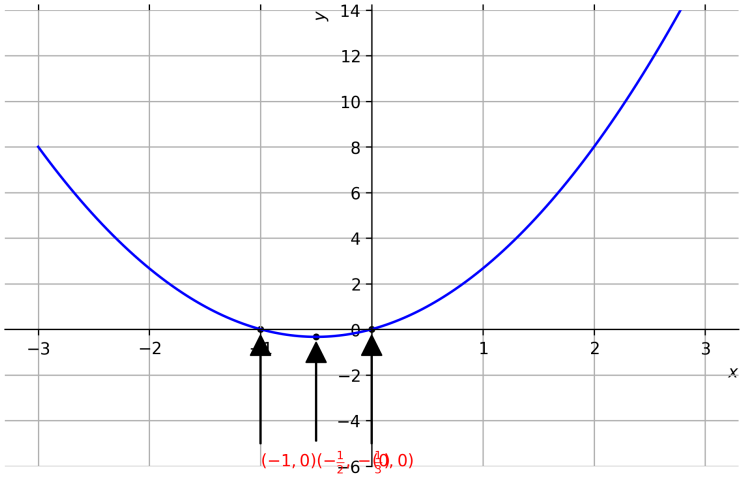
15

2. (a) C
   (b) A
   (c) D
   (d) D
   (e) D
   (f) A
   (g) C
3. (a) $\mathbb{R}$
   (b) (k, ∞)
   (c) $f^{-1}(x) = \frac{\log(-k+x)}{\log(2)}$
   (d) 3
4. (a) $\mathbb{R}$
   (b) -1
   (c) $[-3, \infty)$
   (d) $f^{-1}(x) = \sqrt{x+3} - 1$
5. (a) $\mathbb{R}$
   (b) Amplitude $= 1$, Period $= \frac{2\pi}{3}$
   (c) Maximum $= 4$, Minimum $= 2$
   (d) $x = \frac{2\pi n}{3} + \frac{\pi}{18} + \frac{1}{3}$ or $x = \frac{2\pi n}{3} + \frac{1}{3} + \frac{5\pi}{18}$, $n \in \mathbb{Z}$

6.


y
148
129
110
91
72
53
34
15
−4
−23
−4
−3
−2
−1
2
3
x
(−3, 0)
(0, − 6)
(1, 0)

7.


y
14
12
10
8
6
4
2
0
−2
−4
−6
−3
−2
1
2
3
x
(−1, 0)
(0, 0)