

MATHEMATICAL METHODS

Written examination: SAC

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
7	7	30

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book
- Formula sheet
- Working space is provided throughout the book

Instructions

- Write your **student number** in the space provided above on this page
- Unless otherwise indicated, the diagrams in this book are not drawn to scale
- All written responses must be in English

At the end of the examination

- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1**(7 marks)**a If $f(x)$ is defined as

$$f(x) = (-5x - 4)(5x - 3)$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{9}{4}\right]$$

B Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{29}{4}\right]$$

C Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, -\frac{31}{4}\right]$$

D Domain:

$$\mathbb{R}$$

Range:

$$\left(-\infty, \frac{49}{4}\right]$$

(1 mark)b If $f(x)$ is defined as

$$f(x) = -\frac{3}{-3x - 1}$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\left(-\infty, \frac{53}{3}\right) \cup \left(\frac{53}{3}, \infty\right)$$

Range:

$$(-\infty, -17) \cup (-17, \infty)$$

B Domain:

$$\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$$

Range:

$$(-\infty, 0) \cup (0, \infty)$$

C Domain:

$$\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$$

Range:

$$(-\infty, -7) \cup (-7, \infty)$$

D Domain:

$$\left(-\infty, \frac{41}{3}\right) \cup \left(\frac{41}{3}, \infty\right)$$

Range:

$$(-\infty, -10) \cup (-10, \infty)$$

(1 mark)

c If $f(x)$ is defined as

$$f(x) = 3ax + 5x^2$$

and $g(x)$ is defined as

$$g(x) = 2x - 4$$

solve the equation

$$g(f(x)) = 0$$

A

$$\left[-\frac{3a}{10} - \frac{\sqrt{9a^2 + 380}}{10}, -\frac{3a}{10} + \frac{\sqrt{9a^2 + 380}}{10}\right]$$

B

$$\left[-\frac{3a}{10} - \frac{\sqrt{9a^2 - 220}}{10}, -\frac{3a}{10} + \frac{\sqrt{9a^2 - 220}}{10}\right]$$

C

$$\left[-\frac{3a}{10} - \frac{\sqrt{9a^2 - 80}}{10}, -\frac{3a}{10} + \frac{\sqrt{9a^2 - 80}}{10}\right]$$

D

$$\left[-\frac{3a}{10} - \frac{\sqrt{9a^2 + 40}}{10}, -\frac{3a}{10} + \frac{\sqrt{9a^2 + 40}}{10}\right]$$

(1 mark)

d If

$$f(x) = 5 + \frac{1}{x+1}$$

and

$$g(x) = 4x + 6$$

let

$$h(x) = f(g(x))$$

. What is the range of $h(x)$?

A

$$\left(-\infty, -\frac{7}{4}\right) \cup \left(-\frac{7}{4}, \infty\right)$$

B

$$(-\infty, 5) \cup (5, \infty)$$

C

$$\mathbb{R}$$

D

$$(-\infty, -5) \cup (-5, \infty)$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = \sqrt[3]{-2x-5} + 2$$

what is the maximal range of $f^{-1}(x)$?

A

$$[2, \infty)$$

B

$$(-\infty, 2) \cup (2, \infty)$$

C

$$(-\infty, 2]$$

D

$$\mathbb{R}$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = \frac{5ax+a}{2a+x}$$

find the value of a if $f^{-1}(9) = 1$.

A

$$-9$$

B

$$-8$$

C

$$17$$

D

$$-0.75$$

(1 mark)

g If $f(x)$ is defined as

$$f(x) = \frac{2-x}{5x+3}$$

find $f^{-1}(x)$.

A

$$\left[\frac{16-3x}{5x+8} \right]$$

B

$$\left[\frac{7-3x}{5x-16} \right]$$

C

$$\left[\frac{2(1-18x)}{60x+1} \right]$$

D

$$\left[\frac{2-3x}{5x+1} \right]$$

(1 mark)

Question 2**(4 marks)**a If $f(x)$ is defined as

$$f(x) = \sqrt[3]{5x-2}$$

find $f^{-1}(x)$.

(1 mark)

b Two straight lines intersect at a point $x = 5$. If the equations of those two lines are given below, find the value of x .

$$f(x) = 5a + 3x$$

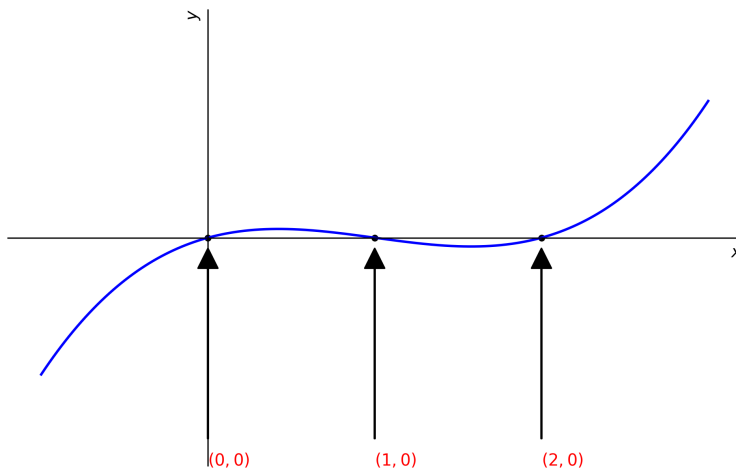
and

$$g(x) = -a - 4x$$

Write your answer correct to 3 decimal places.

(2 marks)

c Consider the following graph.



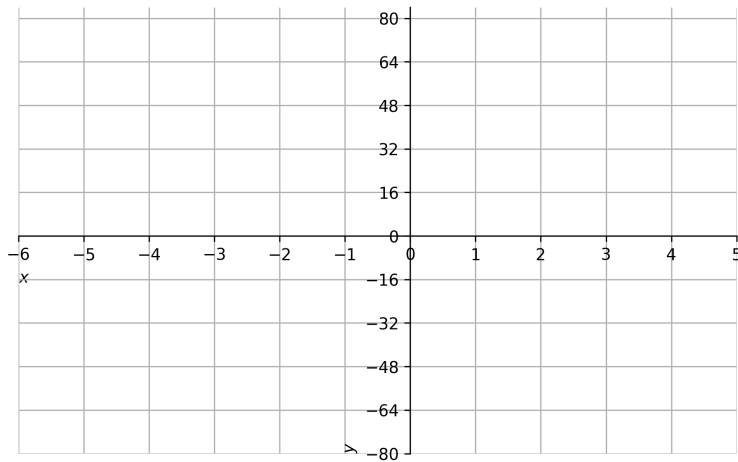
Write down the equation of the cubic function shown in the graph.

(1 mark)

Question 3

Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^3 + 4x^2 - 17x - 60$.

Sketch the graph of $f(x)$ on the following grid.



(3 marks)

Question 4

Consider the function $f(x) = \frac{3x-5}{2x+5}$.

a Find the maximal domain of $f(x)$.

(1 mark)

b Find the range of $f(x)$.

(1 mark)

c Find the asymptotes of $f(x)$.

(2 marks)

Question 5

Consider the function $f : D \rightarrow \mathbb{R}$ where the rule is $f(x) = \sqrt{2-5x}$. D is the maximal domain of the function.

a Find the maximal domain of $f(x)$.

(1 mark)

b Find the range of $f(x)$.

(1 mark)

c Find $f^{-1}(x)$.

(1 mark)

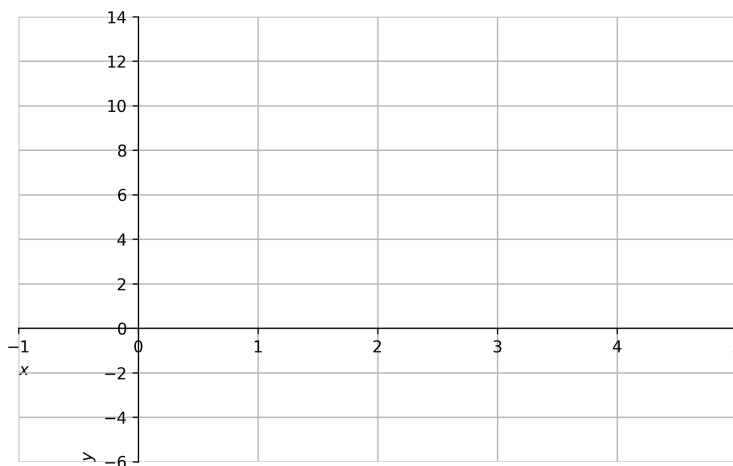
Question 6

A graph is defined by the rule $y = (x - 3)(x - 1)$. The graph is transformed in the following order:

1. vertical dilation by a factor of $\frac{1}{-0.5}$
2. horizontal dilation by a factor of $\frac{1}{0.5}$
3. horizontal translation 2 units to the right

The transformed function is labeled Y .

Sketch the graphs of y and Y in the following grid.



(4 marks)

Question 7

Let $f(x) = x + 5$ and $g(x) = -x^2 + 6x - 1$.

- a State the domain of f . (1 mark)

- b State the range of f . (1 mark)

- c Find the inverse function $f^{-1}(x)$. (1 mark)

- d Solve the equation $f(x) = g(x)$ and hence find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. (2 marks)

Solutions

1. (a) D
(b) B
(c) D
(d) B
(e) D
(f) D
(g) D

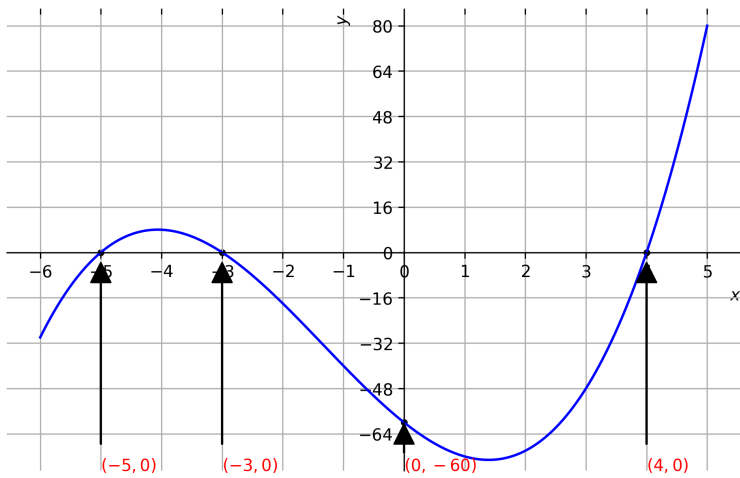
2. (a)

$$\left[\frac{x^3}{5} + \frac{2}{5} \right]$$

- (b) -5.833

(c)

$$x(x-2)(x-1)$$



3.

4. (a)

$$\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$$

(b)

$$\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$

(c)

$$x = [-5/2], y = 3/2$$

5. (a)

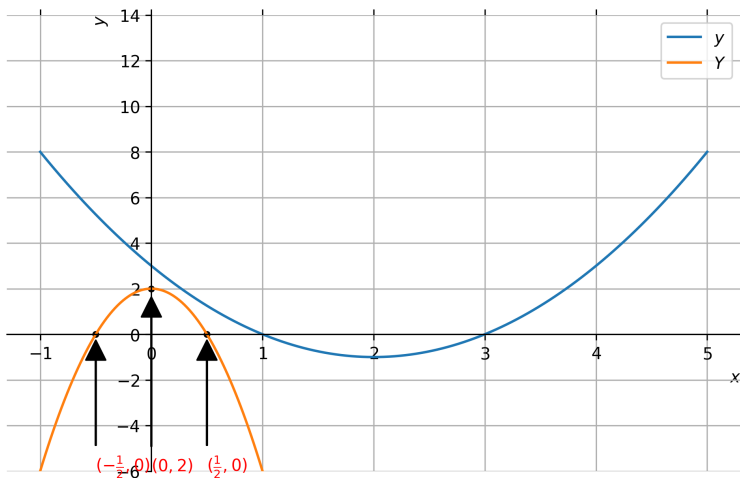
$$\left(-\infty, \frac{2}{5}\right]$$

(b)

$$[0, \infty)$$

(c)

$$\frac{2}{5} - \frac{x^2}{5}$$



6.

7. (a) $\mathbb{R}$

(b) $(-\infty, \infty)$

(c) $f^{-1}(x) = x - 5$

(d) $(2, 7)$ and $(3, 8)$