

# MATHEMATICS ADVANCED

## Written examination: School-based Assessment

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION AND ANSWER BOOK Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| <b>7</b>                   | <b>7</b>                                  | <b>39</b>              |

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

**Question 1****(7 marks)**a Solve the following equation for  $x$ :

$$|x - 6| = 4$$

**(5 marks)**


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b If  $f(x)$  is defined as

$$f(x) = \frac{x-4}{2x-2}$$

find  $f^{-1}(x)$ .**(1 mark)**


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c If  $f(x)$  is defined as

$$f(x) = |x - 6|$$

evaluate  $f(3)$ .**(1 mark)**


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**Question 2****(7 marks)**a If  $f(x)$  is defined as

$$f(x) = 3x + 2$$

what is the maximal domain and range of  $f(x)$ ?

A Domain:

$$\mathbb{R}$$

Range:

$$(-\infty, \infty)$$

B Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{229}{12}, \infty\right)$$

C Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{97}{12}, \infty\right)$$

D Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{145}{12}, \infty\right)$$

(1 mark)

b If  $f(x)$  is defined as

$$f(x) = 5 + \frac{1}{5-x}$$

what is the maximal range of  $f^{-1}(x)$ ?

A

$$(5, \infty)$$

B

$$(-\infty, 5]$$

C

$$(-\infty, 5) \cup (5, \infty)$$

D

$$\mathbb{R}$$

(1 mark)

c If  $f(x)$  is defined as

$$f(x) = -3x - 1, x > 2$$

what is the range of  $f(x)$ ?

A

$$(-\infty, -320)$$

B

$$(-\infty, -161)$$

C

$$(-\infty, -123)$$

D

$$(-\infty, -7)$$

(1 mark)

d If

$$f(x) = -2 + \frac{1}{x+5}$$

and

$$g(x) = -3x - 1$$

let

$$h(x) = f(g(x))$$

. What is the domain of  $h(x)$ ?

A

$$\left(-\infty, -\frac{4}{3}\right) \cup \left(-\frac{4}{3}, \infty\right)$$

B

$$\left(-\infty, \frac{4}{3}\right) \cup \left(\frac{4}{3}, \infty\right)$$

C

$$(-\infty, -2) \cup (-2, \infty)$$

D

$$\mathbb{R}$$

(1 mark)

e If  $f(x)$  is defined as

$$f(x) = 5x - 2$$

and  $g(x)$  is defined as

$$g(x) = -4x - 5$$

solve the equation

$$g(f(x)) + f(g(x)) = 2x$$

A

$$\frac{3}{4}$$

B

$$-\frac{4}{7}$$

C

$$\frac{8}{25}$$

D

$$\frac{40}{43}$$

(1 mark)

f If  $f(x)$  is defined as

$$f(x) = \sqrt[3]{4-x}$$

find  $f^{-1}(x)$ .

A

$$[4 - 125x^3]$$

B

$$[(x-9)^3 + 4]$$

C

$$[4 - x^3]$$

D

$$\left[4 - \frac{x^3}{729}\right]$$

(1 mark)

g If  $f(x)$  is defined as

$$f(x) = \sqrt[4]{3-3x}$$

find  $f^{-1}(x)$ .

A

$$\left[1 - \frac{(x-16)^4}{3}\right]$$

B

$$\left[1 - \frac{x^4}{3}\right]$$

C

$$\left[1 - \frac{x^4}{1875}\right]$$

D

$$\left[1 - \frac{160000x^4}{3}\right]$$

(1 mark)

**Question 3**

Let  $f(x) = 3^x + k$  and  $g(x) = k + x$ , where  $k$  is a constant.

a State the domain of  $f$ .

(1 mark)

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b State the range of  $f$ .

(1 mark)

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c Find the inverse function  $f^{-1}(x)$  in terms of  $k$ .

(2 marks)

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d Given that  $f^{-1}(13) = 2$ , find  $k$ .

(2 marks)

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**Question 4**

Let  $f : D = \{x \in \mathbb{R} : x \leq p\} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 - 8x + 13$ , where  $p$  is a constant.

- a State the maximal domain of the rule for  $f$  before the restriction is applied. (1 mark)

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- b Find the value of  $p$  such that  $f$  is one-to-one on  $D$  and therefore has an inverse. (2 marks)

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- c Hence state the range of  $f$  on this restricted domain. (1 mark)

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- d Find the inverse function  $f^{-1}(x)$ . (2 marks)

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**Question 5**

Consider the function  $f : D \rightarrow R$  where the rule is  $f(x) = 2 - 2 \tan(5x + 2)$ .

- a State the maximal domain of  $f$ . (1 mark)

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- b State the period of  $f$ . (1 mark)

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c State the range of  $f$ .

(1 mark)

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d Solve the equation  $f(x) = 4$  for  $x \in \mathbb{R}$ .

(3 marks)

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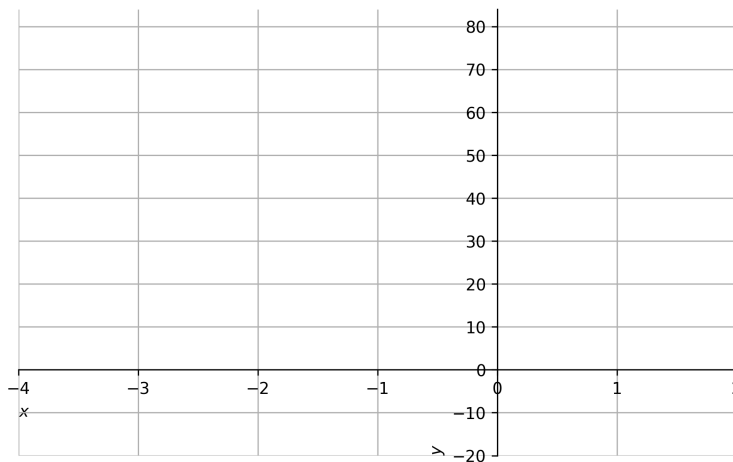


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### Question 6

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^4 + 6x^3 + 9x^2 - 4x - 12$ .

Sketch the graph of  $f(x)$  on the following grid.



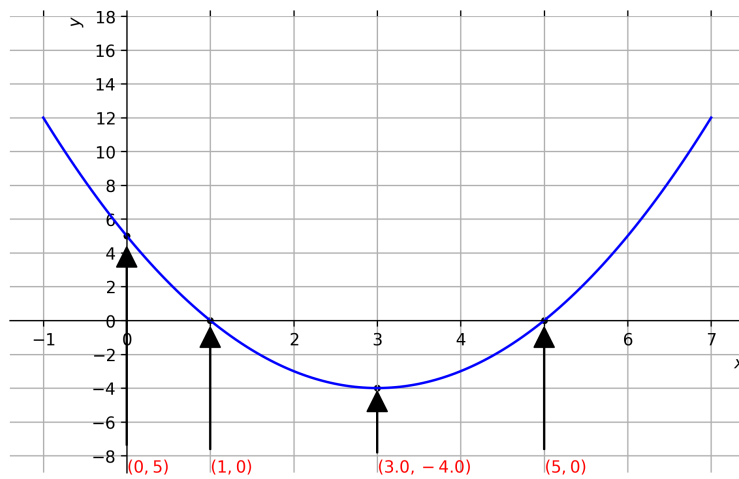
(4 marks)

### Question 7

The parabola below is transformed in the following order:

1. horizontal dilation by a factor of 2
2. vertical dilation by a factor of  $\frac{1}{-3}$
3. horizontal translation 2 units to the right

Sketch the transformed graph in the following grid.



(3 marks)

## Solutions

1. (a) 2 and 10  
(b)

$$\left[ \frac{2(x-2)}{2x-1} \right]$$

(c) 3

2. (a) A  
(b) C  
(c) D  
(d) B  
(e) B  
(f) C  
(g) B

3. (a)  $\mathbb{R}$   
(b)  $(k, \infty)$   
(c)  $f^{-1}(x) = \frac{\log(-k+x)}{\log(3)}$   
(d) 4

4. (a)  $\mathbb{R}$   
(b) 4  
(c)  $[-3, \infty)$   
(d)  $f^{-1}(x) = 4 - \sqrt{x+3}$

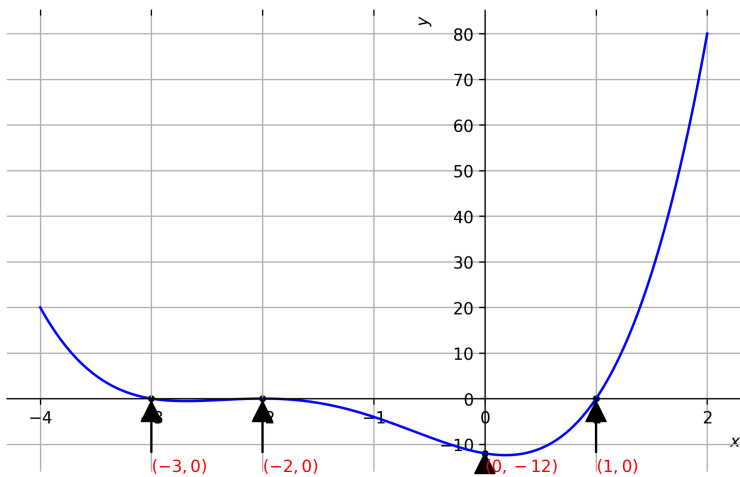
5. (a)

$$\mathbb{R} \setminus \left\{ \frac{\pi n}{5} - \frac{2}{5} + \frac{\pi}{10} : n \in \mathbb{Z} \right\}$$

- (b)  $\frac{\pi}{5}$   
(c)  $\mathbb{R}$   
(d)  $x = \frac{\pi n}{5} - \frac{2}{5} - \frac{\pi}{20}, n \in \mathbb{Z}$



6.



7.

