MATHEMATICS ADVANCED

Written examination: School-based Assessment

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK
Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
7	**7**	**29**

- Calculators approved by NESA may be used.
- A reference sheet is provided at the back of this paper.
- Write your Centre Number and Student Number at the top of this page.

Question 1

(7 marks)

a If $f(x)$ is defined as

$$f(x) = (x+5)(2x-1)$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{177}{8}, \infty\right)$$

B Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{257}{8}, \infty\right)$$

C Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{249}{8}, \infty\right)$$

D Domain:

$$\mathbb{R}$$

Range:

$$\left[-\frac{209}{8}, \infty\right)$$

(1 mark)

b If $f(x)$ is defined as

$$f(x) = -\frac{5}{2-5x}$$

what is the maximal domain and range of $f(x)$?

A Domain:

$$\left(-\infty, \frac{102}{5}\right) \cup \left(\frac{102}{5}, \infty\right)$$

Range:

$$(-\infty, -7) \cup (-7, \infty)$$

B Domain:

$$\left(-\infty, \frac{47}{5}\right) \cup \left(\frac{47}{5}, \infty\right)$$

Range:

$$(-\infty, -20) \cup (-20, \infty)$$

C Domain:

$$\left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$$

Range:

$$(-\infty, 0) \cup (0, \infty)$$

D Domain:

$$\left(-\infty, \frac{62}{5}\right) \cup \left(\frac{62}{5}, \infty\right)$$

Range:

$$(-\infty, -7) \cup (-7, \infty)$$

(1 mark)

c If $f(x)$ is defined as

$$f(x) = 3ax - 4x^2$$

and $g(x)$ is defined as

$$g(x) = 4 - 2x$$

solve the equation

$$g(f(x)) = 0$$

A

$$\left[\frac{3a}{8} - \frac{\sqrt{9a^2+272}}{8}, \frac{3a}{8} + \frac{\sqrt{9a^2+272}}{8}\right]$$

B

$$\left[\frac{3a}{8} - \frac{\sqrt{9a^2-32}}{8}, \frac{3a}{8} + \frac{\sqrt{9a^2-32}}{8}\right]$$

C

$$\left[\frac{3a}{8} - \frac{\sqrt{9a^2-128}}{8}, \frac{3a}{8} + \frac{\sqrt{9a^2-128}}{8}\right]$$

D

$$\left[\frac{3a}{8} - \frac{3\sqrt{a^2-16}}{8}, \frac{3a}{8} + \frac{3\sqrt{a^2-16}}{8}\right]$$

(1 mark)

d If

$$f(x) = -5 + \frac{1}{x-6}$$

and

$$g(x) = 4x - 6$$

let

$$h(x) = f(g(x))$$

. What is the range of $h(x)$?

A

$$\mathbb{R}$$

B

$$(-\infty, 3) \cup (3, \infty)$$

C

$$(-\infty, 5) \cup (5, \infty)$$

D

$$(-\infty, -5) \cup (-5, \infty)$$

(1 mark)

e If $f(x)$ is defined as

$$f(x) = \sqrt[3]{3}\sqrt[3]{-x} + 5$$

what is the maximal range of $f^{-1}(x)$?

A

$$\mathbb{R}$$

B

$$(-\infty, 5]$$

C

$$(-\infty, 5) \cup (5, \infty)$$

D

$$[5, \infty)$$

(1 mark)

f If $f(x)$ is defined as

$$f(x) = \frac{2ax - 5a}{-a + x}$$

find the value of a if $f^{-1}(2) = 10$.

A

$$5$$

B

$$1.176$$

C

$$9$$

D

$$-16$$

(1 mark)

g If $f(x)$ is defined as

$$f(x) = \frac{-5x - 1}{2x + 1}$$

find $f^{-1}(x)$.

A

$$\left[\frac{6 - x}{2x - 15}\right]$$

B

$$\left[\frac{-x - 1}{2x + 5}\right]$$

C

$$\left[\frac{-x - 3}{2x + 15}\right]$$

D

$$\left[\frac{-19x - 1}{38x + 5}\right]$$

(1 mark)

Question 2

(2 marks)

a If $f(x)$ is defined as

$$f(x) = \sqrt[3]{2-x}$$

find $f^{-1}(x)$. (1 mark)

b If $f(x)$ is defined as

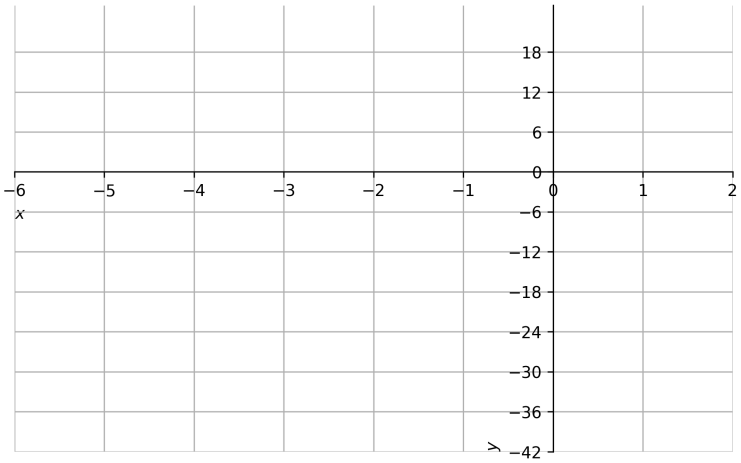
$$f(x) = |2x-6|$$

evaluate $f(2)$. (1 mark)

Question 3

Let $f : R \to R$, $f(x) = x^3 + 5x^2 - x - 5$.

Sketch the graph of $f(x)$ on the following grid.



(3 marks)

Question 4

Consider the function $f : D \to R$ where the rule is $f(x) = \frac{-2x-3}{5x+4}$. D is the maximal domain of the function.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

c For the line defined by the function

$$g(x) = -4ax + 3$$

what values of a will ensure that these two graphs intereset once? (3 marks)

Question 5

Consider the function $f : D \to R$ where the rule is $f(x) = \sqrt{x+3}$. D is the maximal domain of the function.

a Find the maximal domain of $f(x)$. (1 mark)

b Find the range of $f(x)$. (1 mark)

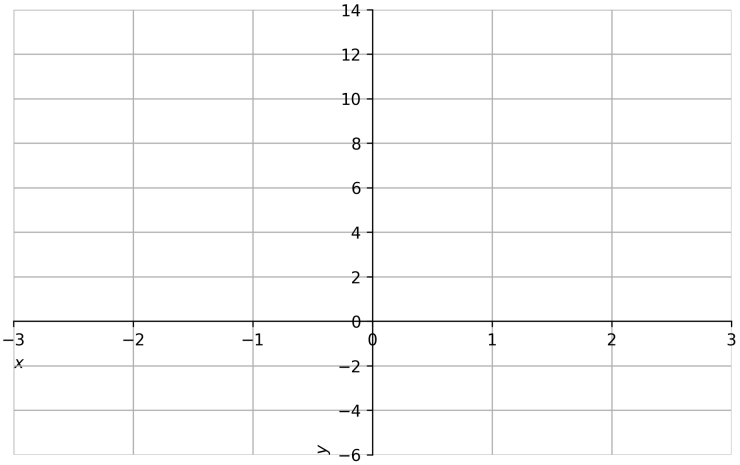
c Find $f^{-1}(x)$. (1 mark)

Question 6

A graph is defined by the rule $y = x^2 - 1$. The graph is transformed in the following order:

1. horizontal dilation by a factor of 3
2. vertical dilation by a factor of $\frac{1}{\frac{1}{2}}$
3. horizontal translation 2 units to the right

The transformed function is labeled Y.
Sketch the graphs of y and Y in the following grid.



(4 marks)

Question 7

Let $f(x) = 3x - 3$ and $g(x) = -x^2 + 9x - 11$.

a State the domain of f. (1 mark)

b State the range of f. (1 mark)

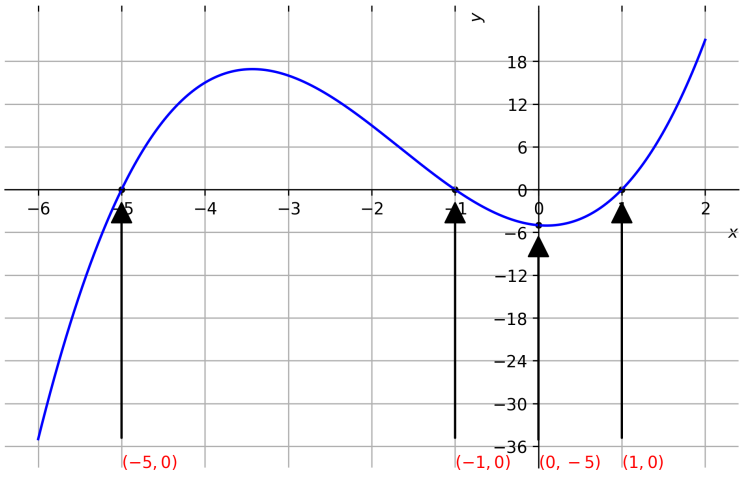
c Find the inverse function $f^{-1}(x)$. (1 mark)

d Solve the equation $f(x) = g(x)$ and hence find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. (2 marks)

Solutions

1. (a) B
   (b) C
   (c) B
   (d) D
   (e) A
   (f) B
   (g) B

2. (a)
   $$[2 - x^3]$$
   (b) 2

3. 

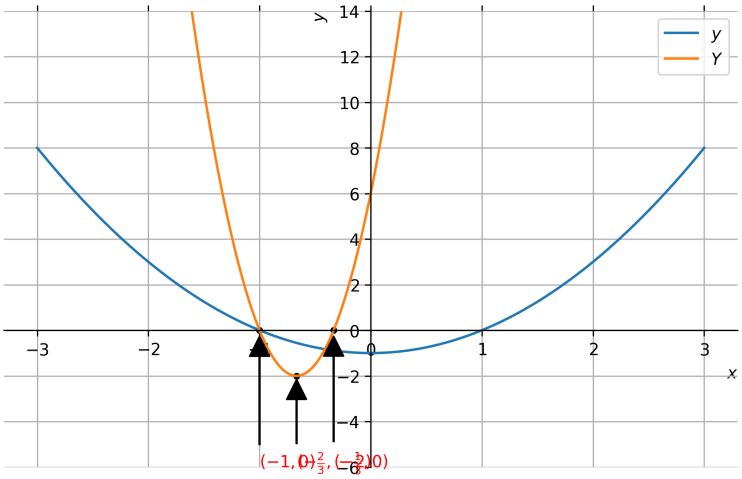
4. (a)
   $$\left(-\infty, -\frac{4}{5}\right) \cup \left(-\frac{4}{5}, \infty\right)$$
   (b)
   $$\left(-\infty, -\frac{2}{5}\right) \cup \left(-\frac{2}{5}, \infty\right)$$
   (c)
   $$-\frac{41}{32} - \frac{5\sqrt{21}}{32}$$

5. (a)
   $$[-3, \infty)$$

(b) $[0, \infty)$

(c) $x^2 - 3$

6.



7. (a) $\mathbb{R}$
   (b) $(-\infty, \infty)$
   (c) $f^{-1}(x) = \frac{x}{3} + 1$
   (d) $(2,3)$ and $(4,9)$